



Name : .....

Total Marks = 466

Time : 8 hr

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- Q1.** Show that  $f: N \rightarrow N$ , given by  $f(x) = \begin{cases} x + 1, & \text{if } x \text{ is odd} \\ x - 1, & \text{if } x \text{ is even} \end{cases}$ , is both one-one and onto. 4
- Q2.** Let  $A$  be the set of all 46 students of class XII in a school. Let  $f: N \rightarrow N$  be a function defined  $f(x) =$  Roll number of the student  $x$ . Show that ' $f$ ' is one-one but not onto. 4
- Q3.** Show that the function  $f: \{0\} \rightarrow R - \{0\}$  defined by  $f(x) = \frac{1}{x}$  is one-one. is the result true, if the domain  $R - \{0\}$  is replaced by  $N$ ? 4
- Q4.** Solve for  $x$  4  

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}, -1 < x < 1$$
- Q5.** Solve for  $x$ , 4  

$$\tan^{-1}\frac{x}{2} + \tan^{-1}\frac{x}{3} = \frac{\pi}{4}; 0 < x < \sqrt{6}$$
- Q6.** Show that the function  $f: R - \{3\} \rightarrow R - \{1\}$  given by  $f(x) = \frac{x-2}{x-3}$  is an onto function. 4
- Q7.** Let  $f: N - \{1\} \rightarrow N$  be defined by  $f(n) =$  the highest prime factor of  $n$  4  
 Show that  $f$  is neither one-one nor onto. Find the range of  $f$ .
- Q8.** Let  $A = R - \{2\}$  and  $B = R - \{1\}$  If  $f: A \rightarrow B$  is a mapping defined by  $f(x) = \frac{x-1}{x-2}$ , show that  $f$  is bijective. 4
- Q9.** If  $f: R \rightarrow R$  be the function defined by  $f(x) = 4x^3 + 7$ , show that  $f$  is a bijection. 4
- Q10.** If functions ' $f$ ' and ' $g$ ' are given by  $f = \{(1, 2), (3, 5), (4, 1), (2, 6)\}$  and  $g = \{(2, 6), (5, 4), (1, 3), (6, 1)\}$ , find the range of ' $f$ ' and ' $g$ ' and write down the functions  $f \circ g$  and  $g \circ f$ . 4
- Q11.** Let  $A = \{1, 2, 3\}$ ,  $B = \{4, 5\}$ ,  $C = \{5, 6\}$ . Let  $f: A \rightarrow B$ ;  $g: B \rightarrow C$  be defined by  $f(1) = 4$ ,  $f(2) = 5$ ,  $f(3) = 4$ ,  $g(4) = 5$ ,  $g(5) = 6$ . Find  $g \circ f: A \rightarrow C$ . 4
- Q12.** If  $f: R \rightarrow R$  is defined by  $f(x) = x^2 - 3x + 2$ , find  $f(f(x))$ . 4
- Q13.** Let  $A = \{x \in R : 0 \leq x \leq 1\}$ . If  $f: A \rightarrow A$  is defined by 4  

$$f(x) = \begin{cases} x, & \text{if } x \in Q \\ 1-x, & \text{if } x \notin Q \end{cases}$$
  
 Then prove that  $f \circ f(x) = x$  for all  $x \in A$ .
- Q14.** Let  $f: R \rightarrow R$  be a function given by  $f(x) = ax + b$  for all  $x \in R$ . Find the constants  $a$  and  $b$  such that  $f \circ f = I_R$ . 4
- Q15.** Consider  $f: N \rightarrow N$ ,  $g: N \rightarrow N$  and  $h: N \rightarrow N$  defined as  $f(x) = 2x$ ,  $g(y) = 3y + 4$  and  $h(z) = \sin z$  for all  $x, y, z \in N$ . Show that  $h \circ (g \circ f) = (h \circ g) \circ f$ . 4

Q16. Find the values of  $a$  and  $b$ . If 4

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \text{ is a matrix satisfying } AA^T = 9I_3,$$

Q17. If  $\sin^{-1} \frac{2a}{1+a^2} + \sin^{-1} \frac{2b}{1+b^2} = 2 \tan^{-1} x$ . Prove that  $x = \frac{a+b}{1-ab}$ . 4

Q18. If  $\cos^{-1} \frac{x}{2} + \cos^{-1} \frac{y}{3} = \alpha$  then prove that  $9x^2 - 12xy \cos \alpha + 4y^2 = 36 \sin^2 \alpha$  4

Q19. Find  $x$ , if 4

$$\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0.$$

Q20. Find  $A$ . If 4

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix},$$

Q21. Consider  $f: R \rightarrow R$  given by  $f(x) = 8x + 5$ . Show that  $f$  is invertible. Find the inverse of  $f$ . 4

Q22. Show that  $f: [-1, 1] \rightarrow R$ , given by  $f(x) = \frac{x}{x+2}$  is one-one, Find the inverse of the function  $f: [-1, 1] \rightarrow \text{Range}(f)$ . 4

Q23. Let  $Y = \{n^2 : n \in N\} \subset N$ . Consider  $f: N \rightarrow Y$  given by  $f(n) = n^2$ . Show that  $f$  is invertible. Find the inverse of  $f$ . 4

Q24. Let  $A = R - \{3\}$  and  $B = R - \{1\}$ . Consider the function  $f: A \rightarrow B$  defined by  $f(x) = \frac{x-2}{x-3}$ . Show that ' $f$ ' is one-one and onto and hence find  $f^{-1}$ . 4

Q25. Let  $f: N \rightarrow Y$  be a function defined as  $f(x) = 4x + 3$ , where  $Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$ . Show that  $f$  is invertible. Find its inverse. 4

Q26. Let  $*$  the binary operation on  $N$  given by  $a * b = \text{l.c.m.}(a, b)$ . Find: 4

(i)  $2 * 3$  (ii)  $5 * 15$  (iii) Is  $*$  commutative? (iv) Find the identity of operation  $*$  in  $N$ . (v) which elements of  $N$  are invertible? Find them.

Q27. A binary operation  $*$  over  $R - \{-1\}$  is defined as  $a * b = \frac{a}{b+1}$ . Is the operation  $*$  commutative, associative? 4

Q28. Let  $*$  be a binary operation defined on  $Q$ . Find which of the binary operations are associative. 4

(i)  $a * b = a - b$  (ii)  $a * b = \frac{ab}{4}$  (iii)  $a * b = a - b + ab$  (iv)  $a * b = ab^2$ .

Q29. Consider the binary operations  $*$ :  $R \times R \rightarrow R$  and  $o$ :  $R \times R \rightarrow R$  defined as  $a * b = |a - b|$  and  $a o b = a$  for all,  $a, b \in R$ . Show that ' $*$ ' is commutative but not associative, ' $o$ ' is associative but not commutative. 4

Q30. Prove that 4

$$2 \tan^{-1} \left( \sqrt{\frac{a-b}{a+b}} \tan \frac{\theta}{2} \right) = \cos^{-1} \left( \frac{a \cos \theta + b}{a + b \cos \theta} \right)$$

- Q31. If:  $\tan^{-1} \left\{ \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right\} = \alpha$ , then prove that  $x^2 = \sin 2\alpha$ . 4
- Q32. If:  $y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$ ,  
 prove that  $\sin y = \tan^2 \frac{x}{2}$  4
- Q33. Show that  $A$  is a root of the polynomial  $f(x)$ . If 4
- $$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}; \quad f(x) = x^3 - 6x^2 + 7x + 2.$$
- Q34. Show that the relation  $R$ , defined on the set  $A$  of all polygons as 4
- $$R = \{(P_1, P_2) : P_1 \text{ and } P_2 \text{ have same number of sides}\},$$
- is an equivalence relation. What is the set of all elements in  $A$  related to the right angle triangle  $T$  with sides 3, 4 and 5?
- Q35. Given a non-empty set  $X$ , consider  $P(X)$  which is the set of all subsets of  $X$ . Define a 4  
 relation in  $P(X)$  as follows:
- For subsets  $A, B$  in  $P(X)$ ,  $A R B$  if  $A \subset B$ . Is  $R$  an equivalence relation on  $P(X)$ ? Justify your answer.
- Q36. Show that the relation  $R$  defined on the set  $A$  of all triangles in a plane as 4
- $$R = \{(T_1, T_2) : T_1 \text{ is similar to } T_2\}$$
- is an equivalence relation.
- Consider three right angle triangles  $T_1$  with sides 3, 4, 5;  $T_2$  with sides 5, 12, 13 and  $T_3$  with sides 6, 8, 10. Which triangles among  $T_1, T_2$  and  $T_3$  are related?
- Q37. Let  $A = \{1, 2, 3, \dots, 9\}$  and  $R$  be the relation in  $A \times A$  defined by  $(a, b) R (c, d)$  if  $a + d = b + c$ , 4  
 for  $(a, b), (c, d) \in A \times A$ . Prove that  $R$  is an equivalence relation, also obtain the equivalence class  $[(2, 5)]$ .
- Q38. In the set of natural numbers  $N$ , define a relation  $R$  as follows: 4
- For every  $m, n \in N$ ,  $n R m$  if on division by 5 each of the integers  $n$  and  $m$  leave the remainder less than 5, i.e., one of the numbers 0, 1, 2, 3, 4. Show that  $R$  is an equivalence relation. Also obtain the pairwise disjoint subsets determined by  $R$ .
- Q39. Check the relation  $R$  for reflexivity, symmetry and transitivity, given as  $a R b$ , if  $b$  is divisible 4  
 by  $a, \forall a, b \in N$ .
- Q40. Let  $f: A \rightarrow B$  be a given function. A relation  $R$  in set  $A$  is given by  $R = \{(a, b) \in A \times A \mid f(a) = f(b)\}$ . 4  
 Check, if  $R$  is an equivalence relation.
- Q41. Let  $P$  be the set of all the points in a plane and the relation  $R$  in set  $P$  be defined as 4  
 $R = \{(A, B) \in P \times P \mid \text{distance between points } A \text{ and } B \text{ is less than } 3 \text{ units}\}$ . Show that the relation  $R$  is not an equivalence relation.
- Q42. Prove that the relation  $R$  in the set  $A = \{1, 2, 3, 4, 5\}$  given  $R = \{(a, b) : |a - b| \text{ is even}\}$ , is an 4  
 equivalence relation.
- Show that all the elements of  $\{1, 2, 3\}$  are related to each other and all the elements of  $\{2, 4\}$  are related to each other, But, no element of  $\{1, 3, 5\}$  is related to any element of  $\{2, 4\}$ .
- Q43. Show that the relation  $R$  defined by  $(a, b) R (c, d) \Rightarrow a + d = b + c$  on the set  $N \times N$  is an 4  
 equivalence relation.

**Q44. Prove that** **4**

$$\text{If } A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}, \text{ then } A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}, \text{ where } n \text{ is any positive integer.}$$

**Q45. Solve for x :** **4**

$$\tan(\cos^{-1} x) = \sin\left(\cot^{-1} \frac{1}{2}\right)$$

**Q46. Solve for x :** **4**

$$3 \sin^{-1} \frac{2x}{1+x^2} - 4 \cos^{-1} \frac{1-x^2}{1+x^2} + 2 \tan^{-1} \frac{2x}{1-x^2} = \frac{\pi}{3}$$

**Q47. Solve for x :** **4**

$$\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$$

**Q48. Solve for x :** **4**

$$\sin^{-1}(1-x) - 2 \sin^{-1} x = \frac{\pi}{2}$$

**Q49. Solve for x :** **4**

$$\tan^{-1} \frac{x-1}{x-2} + \tan^{-1} \frac{x+1}{x+2} = \frac{\pi}{4}$$

**Q50. If  $\sin\left(\sin^{-1} \frac{1}{5} + \cos^{-1} x\right) = 1$ , then find the value of x.** **4**

**Q51. Solve for x,** **4**

$$\cos(2 \sin^{-1} x) = \frac{1}{9}, x > 0$$

**Q52. Show that** **4**

$$\tan\left(\frac{1}{2} \sin^{-1} \frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}.$$

**Q53. Solve following equation for x,** **4**

$$\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2} \tan^{-1} x; x > 0.$$

**Q54. Prove that** **4**

$$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}.$$

**Q55. Solve for x,** **4**

$$\tan^{-1}\left(\frac{1+x}{1-x}\right) = \frac{\pi}{4} + \tan^{-1} x, 0 < x < 1$$

**Q56. Prove that :** **4**

$$\frac{\alpha^3}{2} \operatorname{cosec}^2\left(\frac{1}{2} \tan^{-1} \frac{\alpha}{\beta}\right) + \frac{\beta^3}{2} \sec^2\left(\frac{1}{2} \tan^{-1} \frac{\beta}{\alpha}\right) = (\alpha + \beta)(\alpha^2 + \beta^2).$$

Q57. Prove that :

4

$$\tan^{-1} \left[ \frac{3 \sin 2\theta}{5 + 3 \cos 2\theta} \right] + \tan^{-1} \left( \frac{1}{4} \tan \theta \right) = \theta.$$

Q58. Prove that

4

$$\tan^{-1} \frac{2ab}{a^2 - b^2} + \tan^{-1} \frac{2xy}{x^2 - y^2} = \tan^{-1} \frac{2\alpha\beta}{\alpha^2 - \beta^2}$$

where  $\alpha = ax - by$  and  $\beta = ay + bx$

Q59. Using elementary transformation, find inverse of matrix:

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$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}.$$

Q60. Prove that

4

$$\sin^{-1} \left( \frac{4}{5} \right) + \sin^{-1} \left( \frac{5}{13} \right) + \sin^{-1} \left( \frac{16}{65} \right) = \frac{\pi}{2}.$$

Q61. Prove that :

4

$$2 \tan^{-1} \frac{1}{5} + \sec^{-1} \frac{5\sqrt{2}}{7} + 2 \tan^{-1} \frac{1}{8} = \frac{\pi}{4}.$$

Q62. If  $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$

4

Prove that  $x^4 + y^4 + z^4 + 4x^2 y^2 z^2 = 2(x^2 y^2 + y^2 z^2 + z^2 x^2)$

Q63. If  $x = \operatorname{cosec} \left\{ \tan^{-1} \left\{ \cos \left( \cot^{-1} \left( \sec \left( \sin^{-1} a \right) \right) \right) \right\} \right\}$  Prove that  $x^2 = 3 - a^2$

4

Q64. Prove that :

4

$$\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}] = \sqrt{\frac{x^2 + 1}{x^2 + 2}}$$

Q65. Prove that :

4

$$\tan^{-1} \frac{63}{16} = \sin^{-1} \frac{5}{13} + \cos^{-1} \frac{3}{5}.$$

Q66. Using elementary transformations, find the inverse of the matrix:

4

$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}.$$

Q67. Prove that :

4

$$\tan^{-1} \left\{ \frac{\sqrt{1 + \cos x} + \sqrt{1 - \cos x}}{\sqrt{1 + \cos x} - \sqrt{1 - \cos x}} \right\} = \frac{\pi}{4} - \frac{x}{2}, \text{ if } \pi < x < \frac{3\pi}{2}$$

Q68. Prove that

4

$$\tan^{-1} \sqrt{x} = \frac{1}{2} \cos^{-1} \left( \frac{1-x}{1+x} \right), x \in (0, 1)$$

Q69. Prove that

4

$$\cos^{-1} \left( \frac{\cos \alpha + \cos \beta}{1 + \cos \alpha \cos \beta} \right) = 2 \tan^{-1} \left( \tan \frac{\alpha}{2} \tan \frac{\beta}{2} \right).$$

Q70. Prove that 4

$$\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right) = \frac{\pi}{4}$$

Q71. Prove that 4

$$\cot^{-1}\left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right] = \frac{x}{2};$$

Q72. Using elementary transformations, find the inverse of the matrix: 4

$$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}.$$

Q73. If  $A, B, C$  are three matrices such that  $A = [x \ y \ z]$  4

$$B = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}, \quad C = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \text{find } A(BC).$$

Q74. Show that  $F(x)F(y) = F(x+y)$ . If 4

$$F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Q75. Using elementary transformations, find the inverse of the matrix: 4

$$\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}.$$

Q76. Prove that the product of matrices 4

$$\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix} \text{ and } \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix} \text{ is the null matrix, when } \theta \text{ and } \phi \text{ differ by an odd multiple of } \frac{\pi}{2}.$$

Q77. Show that  $(aI + bA)^n = a^nI + na^{n-1}bA$ , where  $I$  is the identity matrix of order 2 and  $n \in \mathbb{N}$ . 4

$$\text{If } A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

Q78. Prove that a matrix which is both symmetric as well as skew-symmetric is a null matrix. 4

Q79. Show that the matrix  $B'AB$  symmetric or skew symmetric according as  $A$  is symmetric or skew symmetric. 4

Q80. Let  $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ . Let  $R_1$  be a relation on  $X$  given by  $R_1 = \{(x, y) : x - y \text{ is divisible by } 3\}$  and  $R_2$  another relation on  $X$  given by  $R_2 = \{(x, y) : \{x, y\} \subset \{1, 4, 7\} \text{ or } \{x, y\} \subset \{2, 5, 8\} \text{ or } \{x, y\} \subset \{3, 6, 9\}\}$ . Show that  $R_1 = R_2$ . 6

Q81. Express the following matrix as the sum of a symmetric and a skew symmetric 6

$$\text{matrix; } \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}.$$

Q82. Express  $A$  as sum of two matrices such that one is symmetric and the other is skew symmetric. If 6

$$A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}.$$

Q83. Let  $f(x) = \frac{x}{\sqrt{1+x^2}}$ . Then, show that  $(f \circ f \circ f)(x) = \frac{x}{\sqrt{1+3x^2}}$ . 6

Q84. Let  $R$  be the relation defined on the set  $A = \{1, 2, 3, 4, 5, 6, 7\}$  by  $R = \{(a, b) : \text{both } a \text{ and } b \text{ are either odd or even}\}$ . Show that  $R$  is an equivalence relation. Further, show that all the elements of the subset  $\{1, 3, 5, 7\}$  are related to each other and all the elements of the subset  $\{2, 4, 6\}$  are related to each other, but no element of the subset  $\{1, 3, 5, 7\}$  is related to any element of the subset  $\{2, 4, 6\}$ . 6

Q85. Using elementary transformations, find the inverse of the matrix 6

$$\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Q86. Using elementary transformations, find the inverse of the matrix 6

$$\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

Q87. Solve for  $x$ : 6

$$\sin [2 \cos^{-1} \{ \cot (2 \tan^{-1} x) \}] = 0$$

Q88. Let  $f: N \rightarrow R$  be a function defined as  $f(x) = 4x^2 + 12x + 15$ . Show that  $f: N \rightarrow \text{Range}(f)$  is invertible. Find the inverse of  $f$ . 6

Q89. Using elementary transformations, find the inverse of the matrix 6

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Q90. Show that the function  $f: N \rightarrow N$  given by  $f(n) = n - (-1)^n$  for all  $n \in N$  is a bijection. 6

Q91. If  $f: R - \left\{ \frac{7}{5} \right\} \rightarrow R - \left\{ \frac{3}{5} \right\}$  be defined as  $f(x) = \frac{3x+4}{5x-7}$  and  $g: R - \left\{ \frac{3}{5} \right\} \rightarrow R - \left\{ \frac{7}{5} \right\}$  be defined 6

as  $g(x) = \frac{7x+4}{5x-3}$ . Show that  $g \circ f = I_A$  and  $f \circ g = I_B$ , where  $B = R - \left\{ \frac{3}{5} \right\}$  and  $A = R - \left\{ \frac{7}{5} \right\}$ .

Q92. A trust fund has Rs. 30000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs. 30000 among the two types of bonds. If the trust fund must obtain an annual total interest of 6

(i) Rs. 1800

(ii) Rs. 2000

Q93. If  $A$  and  $B$  are square matrices of the same order such that  $AB = BA$ , then prove by induction that  $AB^n = B^nA$ . Further, prove that  $(AB)^n = A^nB^n$  for all  $n \in N$ . 6

Q94. Prove that

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$$A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}, n \in \mathbb{N}. \text{ If } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Q95. If  $a$  is a non-zero real or complex number. Use the principle of mathematical induction to prove that

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$$\text{If } A = \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}, \text{ then } A^n = \begin{bmatrix} a^n & na^{n-1} \\ 0 & a^n \end{bmatrix} \text{ for every positive integer } n.$$

Q96. Prove that, if

6

$$A_\alpha = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$(i) A_\alpha \cdot A_\beta = A_{\alpha + \beta} \quad (ii) (A_\alpha)^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix} \text{ for every positive integer } n.$$

Q97. Verify that  $A^3 - A^2 - 3A - I = 0$ . If

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$$A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}.$$

Q98. Prove that  $A^2 - 5A + 7I = 0$  use this to find  $A^4$ . If

6

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

Q99. Let

6

$$A = \begin{bmatrix} 0 & -\tan(\alpha/2) \\ \tan(\alpha/2) & 0 \end{bmatrix} \text{ and } I \text{ be the identity matrix of order 2. show that}$$

$$I + A = (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

Q100 Show that the relation  $S$  in the set  $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$  given by  $S = \{(a, b) : a, b \in \mathbb{Z}, |a - b| \text{ is divisible by } 4\}$  is an equivalence relation. Find the set of all elements related to 1.

6

Q101 Show that the relation  $R$  on the set  $A$  of points in a plane, given by

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$$R = \{(P, Q) : \text{Distance of the point } P \text{ from the origin is same as the distance of the point } Q \text{ from the origin}\},$$

is an equivalence relation.

Further show that the set of all points related to a point  $P \neq (0, 0)$  is the circle passing through  $P$  with origin as centre.

Q102 If  $f, g : \mathbb{R} \rightarrow \mathbb{R}$  are defined respectively by  $f(x) = x^2 + 3x + 1$ ,  $g(x) = 2x - 3$ , find (i)  $f \circ g$  (ii)  $g \circ f$  (iii)  $f \circ f$  (iv)  $g \circ g$ .

6

Q103 Let,  $f : \mathbb{Z} \rightarrow \mathbb{Z}$  be defined by  $f(n) = 3n$  for all  $n \in \mathbb{Z}$  and  $g : \mathbb{Z} \rightarrow \mathbb{Z}$  be defined by

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$$g(n) = \begin{cases} \frac{n}{3}, & \text{if } n \text{ is a multiple of } 3 \\ 0, & \text{if } n \text{ is not a multiple of } 3 \end{cases} \quad \text{for all } n \in \mathbb{Z}.$$

Show that  $g \circ f = I_{\mathbb{Z}}$  and  $f \circ g \neq I_{\mathbb{Z}}$ .



**Q104** Show that the function  $f : \mathbb{R} \rightarrow \{x \in \mathbb{R} : -1 < x < 1\}$  defined by  $f(x) = \frac{x}{1+|x|}$ ,  $x \in \mathbb{R}$  is one-one function. 6

