



Name : .....

Time : 4 hr

Total Marks = 101

Date: 4/10/2017

**Q1. What is the value of determinant**

**1**

$$\begin{vmatrix} 2x + 4 & 4 \\ x + 5 & 3 \end{vmatrix}.$$

**Q2. Write the value of the determinant**

**1**

$$\begin{bmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{bmatrix}.$$

**Q3. Write value of x. If**

**1**

$$\begin{vmatrix} x + 1 & x - 1 \\ x - 3 & x + 2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}.$$

**Q4. Show that the points (1, 0), (6, 0), (0, 0) are collinear.**

**1**

**Q5. Find the cofactor of  $a_{22}$  and  $a_{32}$  in the following:**

**1**

$$\begin{vmatrix} 3 & 4 & 6 \\ -4 & 3 & -1 \\ 5 & 2 & 8 \end{vmatrix}$$

**Q6. Find the minor of the element of second row and third column in the following determinant**

**1**

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

**Q7. A matrix A of order  $3 \times 3$  has determinant 5. What is the value of  $|4A|$ ?**

**1**

**Q8. If  $|A| = 2$ , where A is a  $2 \times 2$  matrix, find  $|\text{adj } A|$ .**

**1**

**Q9. If A is a square matrix of order 3 such that  $|\text{adj } A| = 64$ , find  $|A|$  [ $\therefore |\text{adj } A| = |A|^{n-1}$ ]**

**1**

**Q10. Find for what value of x the matrix**

**1**

$$\begin{bmatrix} 3 - 2x & x + 1 \\ 2 & 4 \end{bmatrix} \text{ is singular?}$$

**Q11. For what value of x,**

**1**

$$A = \begin{bmatrix} 2(x + 1) & 2x \\ x & x - 2 \end{bmatrix} \text{ is a singular matrix?}$$

- Q12. If  $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$  find  $\frac{dy}{dx}$  1
- Q13. The money to be spent for the welfare of the employees of a firm is proportional to the rate of change of its total revenue (marginal revenue). If the total revenue (in Rs.) received from the sale of  $x$  units of a product is given by  $R(x) = 3x^2 + 36x + 5$ , find the marginal revenue, when  $x = 5$ . 1
- Q14. A stone is dropped into a quiet lake and waves move in a circle at a speed of 3.5 cm/sec. At the instant when the radius of the circular wave is 7.5 cm, how fast is the enclosed area increasing? 1
- Q15. Find an angle  $\theta$ , which increase twice as fast as its sine. 1
- Q16. A balloon which always remain spherical, has a variable diameter  $\frac{3}{2}(2x + 3)$ . Determine the rate of change of volume with respect to  $x$ . 1
- Q17. If  $y = \tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$  find  $\frac{dy}{dx}$  1
- Q18. Find the slope of tangent to the curve  $f(x) = 3x^2 + 4x$  at point, where  $x$ -coordinate is  $-2$ . 1
- Q19. A balloon, which always remains spherical, has a variable radius. Find the rate at which it's volume is increasing with respect to it's radius when the radius is 7 cm. 1
- Q20. Find the rate of change of the volume of a sphere with respect to it's surface area when the radius is 2 cm. 1
- Q21. If  $y = \cos^{-1}\left(\frac{\cos x + \sin x}{\sqrt{2}}\right)$  find  $\frac{dy}{dx}$  1
- Q22. Find the values of  $a$  for which function 2
- $$f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases} \text{ is continuous at } x = 0.$$
- Q23. Discuss the continuity of the function 2
- $$f(x) = \begin{cases} \frac{x^2 - x - 6}{x^2 - 2x - 3} & \text{if } x \neq 3 \\ \frac{5}{8} & \text{if } x = 3 \end{cases} \text{ at the point } x = 3.$$
- Q24. For what value of  $k$ , the following function is continuous at  $x = 0$ ? 2
- $$f(x) = \begin{cases} \frac{1 - \cos 4x}{8x^2} & \text{if } x \neq 0 \\ k & \text{if } x = 0 \end{cases}$$
- Q25. If  $y = \tan^{-1}\left\{\sqrt{\frac{1+\sin x}{1-\sin x}}\right\}$  find  $\frac{dy}{dx}$  2
- Q26. Determine the value of  $k$  for which function 2
- $$f(x) = \begin{cases} \frac{\sin 2x}{5x}, & x \neq 0 \\ k, & x = 0 \end{cases} \text{ is continuous at } x = 0.$$

Q27. show that

2

$$A^{-1} = \frac{1}{19}A. \text{ If } A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}.$$

Q28. What positive value of  $x$  makes following pair of determinants equal?

2

$$\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}, \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}.$$

Q29. Evaluate:

2

$$\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}.$$

Q30. If  $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots \text{to } \infty}}}$ , prove that  $\frac{dy}{dx} = \frac{\cos x}{2y - 1}$

2

Q31. If  $A$  is a non-singular matrix of order 3 and  $|\text{adj } A| = |A|^k$ , then what is value of  $k$ ?

2

Q32. Differentiate  $\log(1 + x^2)$  w.r.t.  $\tan^{-1} x$ .

2

Q33. If  $y = \sin \log x$ , prove that  $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$ .

2

Q34. If  $y = ae^{2x} + be^{-x}$ , prove that  $\frac{d^2 y}{dx^2} - \frac{dy}{dx} - 2y = 0$ .

2

Q35. Discuss the applicability of *Rolle's* theorem for the function  $f(x) = 3 + (x - 2)^{\frac{2}{3}}$  on the interval  $[1, 3]$ .

2

Q36. Examine if *Rolle's* theorem is applicable on the function  $f(x) = [x]$  for  $x \in [5, 9]$ . Can you say something about the converse of *Rolle's* theorem from the function?

2

Q37. If  $y = \sin^{-1}\left(\sqrt{\frac{1}{1+x^2}}\right)$  find  $\frac{dy}{dx}$

2

Q38. If  $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$  find  $\frac{dy}{dx}$

2

Q39. If  $A_{ij}$  is the cofactor of the element  $a_{ij}$  of the determinant

2

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}, \text{ then write the value of } a_{32} \times A_{32}.$$

Q40. The radius of a cylinder is increasing at the rate of 2 cm/sec. and its altitude is decreasing at the rate of 3 cm/sec. Find the rate of change of volume when radius is 3 cm and altitude 5 cm.

2

Q41. A particle moves along the curve  $y = x^2 + 2x$ . At what point(s) on the curve are the  $x$  and  $y$  coordinates of the particle changing at the same rate?

2

Q42. A balloon which always remains spherical, is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon is increasing when the radius is 15 cm.

2

Q43. An edge of a variable cube is increasing at the rate of 10 cm/sec. How fast the volume of the cube is increasing when the edge is 5 cm long?

2

- Q44. If  $y = (x^x)^x$ , find  $\frac{dy}{dx}$ . 2
- Q45. Write the value of  $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$  2
- Q46. If  $xy + y^2 = \tan x + y$ , find  $\frac{dy}{dx}$  2
- Q47. Prove that the function  $f$  given by  $f(x) = \log \sin x$  is strictly increasing on  $\left(0, \frac{\pi}{2}\right)$  and strictly decreasing on  $\left(\frac{\pi}{2}, \pi\right)$ . 2
- Q48. Show that the function  $f(x) = \frac{3}{x} + 7$  is decreasing for  $x \in R (x \neq 0)$ . 2
- Q49. Prove that the function  $f$  given by  $f(x) = x^3 - 3x^2 + 4x$  is strictly increasing on  $R$ . 2
- Q50. Find the values of  $b$  for which the function  $f(x) = \sin x - bx + c$  is decreasing function on  $R$ . 2
- Q51. Find the least value of  $a$  such that the function  $x^2 + ax + 1$  is increasing on  $[1, 2]$ . 2
- Q52. Find the equation of the tangent to the curve  $y = -5x^2 + 6x + 7$  at the point  $\left(\frac{1}{2}, \frac{35}{4}\right)$ . 2
- Q53. Find the slope of the normal to the curve  $x = a \cos^3 \theta$ ,  $y = a \sin^3 \theta$  at  $\theta = \frac{\pi}{4}$ . 2
- Q54. The slope of the curve  $2y^2 = ax^2 + b$  at  $(1, -1)$  is  $-1$ . Find  $a, b$ . 2
- Q55. Prove that the tangents to the curve  $y = x^2 - 5x + 6$  at the points  $(2, 0)$  and  $(3, 0)$  are at right angles. 2
- Q56. If  $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \dots \infty}}}$ , prove that  $(2y - 1) \frac{dy}{dx} = \frac{1}{x}$  2
- Q57. If the radius of a sphere is measured as 9m with an error of 0.03m, find the approximate error in calculating its surface area. 2
- Q58. Find the approximate value of  $f(5.001)$ , where  $f(x) = x^3 - 7x^2 + 15$ . 2
- Q59. Find the approximate change in the volume  $V$  of a cube of side  $x$  meters caused by increasing the side by 2%. 2
- Q60. Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals. 2
- $f(x) = (x - 1)^2 + 3$  in  $[-3, 1]$
- Q61. If the function  $x^4 - 62x^2 + ax + 9$  attains the maximum value at  $x = 1$  on the interval  $[0, 2]$ . Find the value of  $a$ . 2