V-JEE INSTITUTE

MATHEMATICS - XII

Vector Algebra, Three Dimentional Geometry, Linear Programming

Name :..... Total Marks = 88

Time : 6 hr Date: 23/10/2017

Q1. Write the position vector of mid-point of the vector joining points $A(3, 4, -2)$ and $B(1, 2, 4)$. 1

Q2. If A, B and C are the vertices of a ΔABC, then what is the value of $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$? 1

Q3. Find the scalar components of $\overrightarrow{AB}$ with initial point $\vec{A}$ $(2, 1)$ and terminal point $\vec{B}$ $(-5, 7)$. 1

Q4. Find the angle between $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively and when $|\vec{a} \times \vec{b}| = \sqrt{3}$. 1

Q5. Write the angle between vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively, having $\vec{a}.\vec{b} = \sqrt{6}$. 1

Q6. Find the length of the perpendicular drawn from the origin to the plane $2x - 3y + 6z + 21 = 0$. 1

Q7. If $|\vec{a}| = \sqrt{3}, |\vec{b}| = 2$ and $\vec{a}.\vec{b} = 3$, find angle between $\vec{a}$ and $\vec{b}$. 1

Q8. If the line $\vec{r} \cdot (\hat{i} - 2\hat{j} + \hat{k}) + \lambda(2\hat{i} + \hat{j} + 2\hat{k})$ is parallel to the plane $\vec{r} \cdot (3\hat{i} - 2\hat{j} + m\hat{k}) = 14$, find the value of m. 1

Q9. If $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a}.\vec{b} = 3$, find the projection of $\vec{b}$ on $\vec{a}$. 1

Q10. Find the equation of plane passing through the point $\hat{i} + \hat{j} + \hat{k}$ and parallel to the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + 2\hat{k}) = 5$. 1

Q11. Show that the planes $2x + 6y + 6z = 7$ and $3x + 4y - 5z = 8$ are at right angles. 1

Q12. Find sum of vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$. 1

Q13. Write a unit vector in the direction of the sum of vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + 3\hat{k}$. 1

Q14. If $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$ and $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$ are two equal vectors, then write the value of $x + y + z$. 1

Q15. Find the magnitude of the vector $\vec{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}$. 1

Q16. Find a normal vector to the plane $2x - y + 2z = 5$. Also, find a unit vector normal to the plane. 1

Q17. Find the vector equation of a plane which is at a distance of 8 units from the origin and which is normal to the vector $2\hat{i} + \hat{j} + 2\hat{k}$. 1

Q18. Find the distance of point $(2, 3, 4)$ from x-axis. 1

Q19. Write the equation of the plane whose intercepts on the coordinate axes are -4, 2 and 3. 1

Q20. Find the value of λ, such that the line $\frac{x-2}{6}=\frac{y-1}{\lambda}=\frac{z+5}{-4}$ is perpendicular to the plane $3x-y-2z=7$. 1

Q21. If a unit vector $\hat{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find the value of θ. 1

Q22. If $P = (1, 5, 4)$ and $Q = (4, 1, -2)$, find the direction ratios of PQ. 1

Q23. If the equation of line AB are $\frac{3-x}{1}=\frac{y+2}{-2}=\frac{z-5}{4}$, write the direction ratios of line parallel to above line AB. 1

Q24. Find the distance between the parallel planes $x + y - z + 4 = 0$ and $x + y - z + 5 = 0$. 1

Q25. Find the value of λ, if the vectors $2\hat{i}+\lambda\hat{j}+3\hat{k}$ and $3\hat{i}+2\hat{j}-4\hat{k}$ are perpendicular to each other. 1

Q26. Write the equation of line parallel to the line $\frac{x-2}{-3}=\frac{y+3}{2}=\frac{z+5}{6}$ and passing through point (1, 2, 3). 1

Q27. Find value of following 2

$\hat{i}\cdot(\hat{j}\times\hat{k})+\hat{j}\cdot(\hat{i}\times\hat{k})+\hat{k}\cdot(\hat{i}\times\hat{j})$.

Q28. Find a vector of magnitude 9, which is perpendicular to both the vectors $4\hat{i}-\hat{j}+3\hat{k}$ and $-2\hat{i}+\hat{j}-2\hat{k}$. 2

Q29. Find a vector in the direction of $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ which has magnitude 6 units. 2

Q30. Find λ when projection of $\vec{a}=\lambda\hat{i}+\hat{j}+4\hat{k}$ on $\vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$ is 4 units. 2

Q31. If $\vec{p}=5\hat{i}+\lambda\hat{j}-3\hat{k}$ and $\vec{q}=\hat{i}+3\hat{j}-5\hat{k}$, then find the value of λ, so that $\vec{p}+\vec{q}$ and $\vec{p}-\vec{q}$ are perpendicular vectors. 2

Q32. Let $\vec{a}=4\hat{i}+5\hat{j}-\hat{k}$, $\vec{b}=\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}-\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and satisfying $\vec{d}.\vec{c}=21$. 2

Q33. If two vectors a and b are such that $|\vec{a}| = 2$ and $|\vec{b}| = 1$ and $\vec{a}.\vec{b} = 1$, then find $(3\vec{a}-5\vec{b}).(2\vec{a}+7\vec{b})$. 2

Q34. If vectors $\vec{a} = 2\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b} = -\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c} = 3\hat{i}+\hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find value of λ. 2

Q35. Prove that the vector normal to the plane containing three points whose positon vectors are $\vec{a},\vec{b},\vec{c}$ lies in the direction $\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}$. 2

Q36. If $\vec{a},\vec{b},\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$. 2

Q37. Prove Cauchy-Schwarz inequality 2

$$(\vec{a}\cdot\vec{b})^2 \le |\vec{a}|^2 |\vec{b}|^2$$

and hence show that $(a_1b_1+ a_2b_2+ a_3b_3)^2 \le (a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)$.

Q38. If $\hat{a}$ and $\hat{b}$ are unit vectors inclined at an angle θ, then prove that $\sin\frac{\theta}{2}=\frac{1}{2}|\hat{a}-\hat{b}|$ 2

Q39. If the points $A(-1, 3, 2)$, $B(-4, 2, -2)$ and $C(5, 5, \lambda)$ are collinear, find the value of λ. 2

Q40. Write the vector equation of the line given by 2

$$\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}.$$

Q41. Find the direction cosines of a line parallel to line. 2

$$\frac{4-x}{2}=\frac{y+3}{2}=\frac{z+2}{1}.$$

Q42. The equation of a line is $\frac{2x-5}{4}=\frac{y+4}{3}=\frac{6-z}{6}$. Find the direction cosines of the line parallel to this line. 2

Q43. $\vec{A}$ and $\vec{B}$ are two points with position vectors $2\vec{a}-3\vec{b}$ and $6\vec{b}-\vec{a}$, respectively. Write the position vector of a point $\vec{P}$ which divides the line segment $\overrightarrow{AB}$ internally in the ratio 1 : 2. 2

Q44. Write the position vector of mid-point of the vector joining points P (2, 3, 4) and Q (4, 1, –2). 2

Q45. Write a unit vector in the direction of $\vec{a}=2\hat{i}-6\hat{j}+3\hat{k}$. 2

Q46. P and Q are two points with position vectors $3\vec{a}-2\vec{b}$ and $\vec{a}+\vec{b}$, respectively. Write the position vector of a point R which divides the line segment PQ in the ratio 2:1 externally. 2

Q47. Find the value of P, if $\left(2\hat{i}+6\hat{j}+27\hat{k}\right)\times\left(\hat{i}+3\hat{j}+P\hat{k}\right)=0$. 2

Q48. Find a unit vector in the direction of vector $\vec{a}=-2\hat{i}+\hat{j}+2\hat{k}$. 2

Q49. Prove that if a plane has the intercepts a, b, c along their respective axes and is at a distance of p units from the origin, then $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$. 2

Q50. If the points (1, 1, λ) and (–3, 0, 1) be equidistant from the plane $\vec{r}\cdot(3\hat{i}+4\hat{j}-12\hat{k})+13=0$, find the value of λ. 2

Q51. Find distance of the plane $3x - 4y + 12z = 3$ from the origin. 2

Q52. Write the intercept cut-off by plane $2x + y – z = 5$ on x-axis. 2

Q53. If $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$ be the position vectors of points A, B, C, D respectively and $\vec{b}-\vec{a}=2(\vec{d}-\vec{c})$, show that the point of intersection of straight lines AD and BC divide these lines in the ratio 2 : 1. 2

Q54. Find $|\vec{x}|$ if $\vec{a}$ is unit vector such that $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=15$. 2

Q55. Find the angle between the normal to the planes $2x – y + z = 6$ and $x + y + 2z = 7$. 2

Q56. Reduce the equation $\vec{r}\cdot(3\hat{i}-4\hat{j}+12\hat{k})=5$ to normal form and hence find the length of perpendicular from the origin to the plane. 2

Q57. If from a point $P(a, b, c)$ perpendiculars PA and PB are drawn to yz and zx-planes, find the vector equation of the plane OAB. 2