Vector Algebra , Three Dimentional Geometry

Name :...................... Total Marks = 526

Time : 35 hr Date: 24/10/2017

Q1. A girl walks 4 km westwards, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure. **4**

Q2. Write all unit vectors of *XY*-plane. **4**

Q3. Find the lengths of the medians of the triangle formed by $A(4, 2)$, $B(1, -2)$ and $C(-2, 6)$ by vector method. **4**

Q4. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors, no two of which are collinear and the vector $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$, then show that $\vec{a}+2\vec{b}+6\vec{c}=\vec{0}$. **4**

Q5. Show that the three points $A(1, -2, -8)$, $B(5, 0, -2)$ and $C(11, 3, 7)$ are collinear, and find the ratio in which B divides AC. **4**

Q6. If $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+3\hat{j}-\hat{k}$, $\vec{c}=-2\hat{i}+\hat{j}-3\hat{k}$, and $\vec{d}=3\hat{i}+2\hat{j}+5\hat{k}$, find scalar α, β and γ such that $\vec{d}=\alpha\vec{a}+\beta\vec{b}+\gamma\vec{c}$. **4**

Q7. If the point $A(2, \beta, 3)$, $B(\alpha, -5, 1)$ and $C(-1, 11, 9)$ are collinear, find the values of α and β by vector method. **4**

Q8. The two adjacent sides of a parallelogram are $2\hat{i}+4\hat{j}-5\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$. Find the unit vectors along the diagonals of the parallelogram. **4**

Q9. A triangle has vertices (1, 2, 4), (– 2, 2, 1) and (2, 4, – 3), prove that the triangle is right-angled and find its other angles. **4**

Q10. The position vectors of the points, *P, Q, R* and *S* are respectively, $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+5\hat{j}$, $3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$, prove that the lines *PQ* and *RS* are parallel and the ratio of their length is $\frac{1}{2}$. **4**

Q11. Find a vector of magnitude 5 units and parallel to the resultant of $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$. **4**

Q12. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=4\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$. Find a vector of magnitude 6 units which is parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$. **4**

Q13. Find the position vector of a point $\vec{R}$ which divides the line joining two points $\vec{P}$ and $\vec{Q}$ whose position vectors are $2\vec{a}+\vec{b}$ and $\vec{a}-3\vec{b}$ respectively, externally in the ratio 1:2. Also show that $\vec{P}$ is the mid-point of line segment $\overrightarrow{RQ}$. **4**

Q14. A vector $\vec{n}$ of magnitude 8 units is inclined to *x*-axis at 45°, *y*-axis at 60° and an acute angle with *z*-axis. If a plane passes through a point $(\sqrt{2}, -1, 1)$ and is normal to $\vec{n}$, find its equation in vector form. **4**

Q15. Find the shortest distance between the lines **4**

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$
$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

Q16. Find the shortest distance between the lines **4**

$$l_1 : \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{1}$$
$$l_2 : \frac{x-2}{2} = \frac{y+1}{1} = \frac{z+1}{2}.$$

Q17. Find the vector equation of the plane which contains the line of intersection of the planes $\vec{r}\cdot(\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$ and $\vec{r}\cdot(2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\vec{r}\cdot(5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$. **4**

Q18. Find the equation of the plane passing through the intersection of the planes $4x - y + z = 10$ and $x + y - z = 4$ and parallel to the line with direction ratios proportional to 2, 1, 1. Find also the perpendicular distance of (1, 1, 1) from this plane. **4**

Q19. Find the shortest distance between lines **4**

$$\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$$
$$\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$$

Q20. State whether the line $\vec{r} = \vec{a} + \lambda\vec{b}$ is parallel to the plane $\vec{r}\cdot\vec{n} = d$. Show that the line $\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} + \hat{j} + 4\hat{k})$ is parallel to the plane $\vec{r}\cdot(-2\hat{i} + \hat{k}) = 5$. Also, find the distance between the line and the plane. **4**

Q21. Show that the line whose vector equation is $\vec{r} = (2\vec{i} - 2\vec{j} + 3\vec{k}) + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ is parallel to the plane $\vec{r}\cdot(\vec{i} + 5\vec{j} + \vec{k}) = 5$. Also find the distance between them. **4**

Q22. Find the equation of the plane passing through the points (1, 0, – 1), (3, 2, 2) and parallel to the line $\frac{x-1}{1} = \frac{y-1}{-2} = \frac{z-2}{3}$. **4**

Q23. Show that the projection vector of $\vec{a}$ on $\vec{b}$ $(\neq 0)$ (component of $\vec{a}$ along $\vec{b}$) is $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$. **4**

Q24. For any two vectors $\vec{a}$ *and* $\vec{b}$, prove that **4**

$|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$ (Triangle inequality)

Q25. For any two vectors $\vec{a}$ and $\vec{b}$, show that : **4**

$(1 + |\vec{a}|^2)\cdot(1 + |\vec{b}|^2) = \{1 - \vec{a}\cdot\vec{b}\}^2 + |\vec{a} + \vec{b} + (\vec{a}\times\vec{b})|^2$.

Q26. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vector such that $\vec{a}\times\vec{b} = \vec{c}$ and $\vec{b}\times\vec{c} = \vec{a}$; prove that $\vec{a}, \vec{b}, \vec{c}$ are mutually at right angles and $|\vec{b}| = 1$ and $|\vec{c}| = |\vec{a}|$. **4**

Q27. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$, $\vec{a}\times\vec{b} = \vec{a}\times\vec{c}$, $\vec{a} \neq \vec{0}$, then show that $\vec{b} = \vec{c}$. **4**

Q28. If $\vec{a}\times\vec{b} = \vec{a}\times\vec{c}$, $\vec{a} \neq \vec{0}$ and $\vec{b} \neq \vec{c}$, show that $\vec{b} = \vec{c} + t\vec{a}$ for some scalar *t*. **4**

Q29. If *A, B, C, D* be any four points in space, prove that **4**

$|\overrightarrow{AB}\times\overrightarrow{CD}+\overrightarrow{BC}\times\overrightarrow{AD}+\overrightarrow{CA}\times\overrightarrow{BD}| = 4$ (Area of triangle *ABC*).

Q30. If $\vec{a},\vec{b},\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then prove that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$ **4**

Q31. If $\vec{a},\vec{b},\vec{c}$ are the position vectors of the vertices *A, B, C* of a triangle *ABC,* show that the area of triangle ABC is $\frac{1}{2}|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|$. **4**

Deduce the condition for points $\vec{a},\vec{b},\vec{c}$ to be collinear.

Q32. Find the area of the triangle whose vertices are *A* (3, – 1, 2), *B* (1, – 1, – 3) and *C* (4, – 3, 1). **4**

Q33. Using vectors find the area of triangle with vertices $\vec{A}$ (1, 1, 2), $\vec{B}$ (2, 3, 5) and $\vec{C}$ (1, 5, 5). **4**

Q34. Using vectors, find the area of triangle with vertices $\vec{A}$ (2, 3, 5), $\vec{B}$ (3, 5, 8) and $\vec{C}$ (2, 7, 8). **4**

Q35. Find the values of *x* for which the angle between the vectors $2x^2\hat{i}+4x\hat{j}+\hat{k}$ and $\vec{b}=7\hat{i}-2\hat{j}+x\hat{k}$ is obtuse. **4**

Q36. Let $\vec{a},\vec{b},\vec{c}$ be unit vectors such that $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}=0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, prove that $\vec{a}=\pm 2(\vec{b}\times\vec{c})$. **4**

Q37. Show that the angle between two diagonal of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$. **4**

Q38. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, show that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$, where $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$. **4**

Q39. Three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}$ = 0. Find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$, if $|\vec{a}|$ = 1, $|\vec{b}|$ = 4 and $|\vec{c}|$ = 2. **4**

Q40. If $\vec{a},\vec{b},\vec{c}$ are three vectors such that $|\vec{a}|$ = 5, $|\vec{b}|$ = 12, $|\vec{c}|$ = 13 and $\vec{a}+\vec{b}+\vec{c}=0$, find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$. **4**

Q41. If *a, b* and *c* are three vectors such that $|\vec{a}|$ = 3, $|\vec{b}|$ = 4 and $|\vec{c}|$ = 5 and each one of these is perpendicular to the sum of other two, find $\vec{a}+\vec{b}+\vec{c}$. **4**

Q42. Find the components of a unit vector which is perpendicular to the vectors $\hat{i}+2\hat{j}-\hat{k}$ and $3\hat{i}-\hat{j}+2\hat{k}$. **4**

Q43. Let $\vec{a}=2\hat{i}+\hat{k}$, $\vec{b}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}=4\hat{i}-3\hat{j}+7\hat{k}$ be three vectors. Find a vector $\vec{r}$ which satisfies $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}\cdot\vec{a}=0$ **4**

Q44. The scalar product of vector $\hat{i}+\hat{j}+\hat{k}$ with the unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ. **4**

Q45. If $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+5\hat{j}$, $3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ respectively are the position vectors of points *A, B, C* and *D*, then find the angle between the straight lines *AB* and *CD*. Prove that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear. **4**

Q46. Let $\vec{a}=\hat{i}-\hat{j}, \vec{b}=3\hat{j}-\hat{k}$ and $\vec{c}=7\hat{i}-\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}\cdot\vec{d}=1$. **4**

Q47. If $\vec{a}=\hat{i}-\hat{j}+7\hat{k}$ and $\vec{b}=5\hat{i}-\hat{j}+\lambda\hat{k}$, then find the value of λ, so that $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular vectors. **4**

Q48. Find a unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}+3\hat{k}$. **4**

Q49. Find a unit vector perpendicular to the plane *ABC* where *A*, *B*, *C* are the points (3, – 1, 2) (1, – 1, – 3), (4, – 3, 1) respectively. **4**

Q50. Find the vector equation of the plane through the points (2, 1, – 1) and (– 1, 3, 4) and perpendicular to the plane $x - 2y + 4z = 10$. **4**

Q51. Find the vector equation of a line passing through a point with position vector $2\hat{i}-\hat{j}+\hat{k}$ and parallel to the line joining the points $-\hat{i}+4\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+2\hat{k}$. Also, find the Cartesian equivalent of this equation. **4**

Q52. A line passes through (2, – 1, 3) and is perpendicular to the line $\vec{r}=(\hat{i}+\hat{j}+\hat{k})+\lambda(2\hat{i}-2\hat{j}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j}-3\hat{k})+\mu(\hat{i}+2\hat{j}+2\hat{k})$. Obtain its equation. **4**

Q53. Find the plane passing through (4, – 1, 2) and parallel to the lines **4**

$$\frac{x+2}{3}=\frac{y-2}{-1}=\frac{z+1}{2} \quad \text{and} \quad \frac{x-2}{1}=\frac{y-3}{2}=\frac{z-4}{3}$$

Q54. Find the coordinates of the point, where the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}$ intersects the plane $x - y + z - 5 = 0$. Also, find the angle between the line and the plane. **4**

Q55. Find the direction cosines of the line $\frac{x-2}{2}=\frac{2y-5}{-3}$, $z = -1$. Also, find the vector equation of the line. **4**

Q56. Find the equation of the plane passing through the point (– 1, 2, 1) and perpendicular to the line joining the points (– 3, 1, 2) and (2, 3, 4). Find also the perpendicular distance of the origin from this plane. **4**

Q57. Find the equation of the plane through the line of intersection of $\vec{r}\cdot(2\hat{i}-3\hat{j}+4\hat{k})=1$ and $\vec{r}\cdot(\hat{i}-\hat{j})+4=0$ and perpendicular to $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})+8=0$. **4**

Q58. Find the points on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance of 5 units from the point *P* (1, 3, 3). **4**

Q59. Find the shortest distance between lines **4**

$$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1} \quad \text{and} \quad \frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}.$$

Q60. Find the shortest distance between lines l_1 and l_2 whose vector equations are given below **4**

$$l_1: \vec{r} = \hat{i}+\hat{j}+\lambda(2\hat{i}-\hat{j}+\hat{k}) \qquad l_2: \vec{r} = 2\hat{i}+\hat{j}-\hat{k}+\mu(3\hat{i}-5\hat{j}+2\hat{k})$$

Q61. Find the shortest distance between the lines **4**

$$\vec{r} = \left(4\hat{i} - \hat{j}\right) + \lambda\left(\hat{i} + 2\hat{j} - 3\hat{k}\right) \quad \text{and} \quad \vec{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \mu\left(2\hat{i} + 4\hat{j} - 5\hat{k}\right).$$

Q62. Find the value of λ, so that the lines **4**

$$\frac{1-x}{3} = \frac{y-2}{2\lambda} = \frac{z-3}{2} \quad \text{and} \quad \frac{x+1}{3\lambda} = \frac{y-1}{1} = \frac{6-z}{7}$$

are perpendicular to each other.

Q63. Prove that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{4} = \frac{z-6}{7}$ are coplanar. Also, find the plane containing these two lines. **4**

Q64. Find the shortest distance between lines whose vector equations are **4**

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}, \quad \vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$$

Q65. Find shortest distance between lines **4**

$$\vec{r} = (1+\lambda)\hat{i} + (2-\lambda)\hat{j} + (\lambda+1)\hat{k}$$

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right).$$

Q66. Find the shortest distance between the lines whose vector equations are **4**

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) \text{ and } \vec{r} = \left(2\hat{i} + 4\hat{j} + 5\hat{k}\right) + \mu\left(4\hat{i} + 6\hat{j} + 8\hat{k}\right)$$

Q67. Find the shortest distance between the lines **4**

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}.$$

Q68. Find the value of λ, so that the lines **4**

$$\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{5z-10}{11} \quad \text{and} \quad \frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$$

are perpendicular to each other.

Q69. Show that the distance d from point P to the line l having equation $\vec{r} = \vec{a} + \lambda\vec{b}$ is given by **4**

$d = \frac{\left|\vec{b} \times \overline{PQ}\right|}{\left|\vec{b}\right|}$, where Q is any point on the line l.

Q70. Find the angle between following pair of lines **4**

$$\frac{-x+2}{-2} = \frac{y-1}{7} = \frac{z+3}{-3} \quad \text{and} \quad \frac{x+2}{-1} = \frac{2y-8}{4} = \frac{z-5}{4}$$

and check whether the lines are parallel or perpendicular.

Q71. Show that the lines $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$; $\vec{r} = 5\hat{i} - 2\hat{j} + \mu\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ are intersecting. Hence, find their point of intersection. **4**

Q72. Find the angle between pair of lines given by **4**

$$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right) \text{ and } \vec{r} = 7\hat{i} - 6\hat{j} - 6\hat{k} + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right).$$

Q73. Show that the following pair of lines do not intersect each other. **4**

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5} \quad \text{and} \quad \frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2}$$

Q74. The points A (4, 5, 10), B (2, 3, 4) and C (1, 2, – 1) are three vertices of parallelogram $ABCD$. Find the vector equations of sides AB and BC and also find coordinates of point D. **6**

Q75. Show that the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ respectively, are the vertices of a right angled triangle. Also, find the remaining angles of the triangle. **6**

Q76. Find the equation of line passing through points A (0, 6, – 9) and B (– 3, – 6, 3). If D is the foot of perpendicular drawn from the point C (7, 4, – 1) on the line AB, then find the coordinates of point D and equation of line CD. **6**

Q77. Find the vector and Cartesian equations of line passing through point (1, 2, – 4) and perpendicular to the lines **6**

$$\frac{x-8}{3} = \frac{y-19}{-16} = \frac{z-10}{7} \quad \text{and} \quad \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}.$$

Q78. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{p}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $p \cdot c = 18$. **6**

Q79. Find the distance of point (– 2, 3, – 4) from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$. **6**

Q80. Find the distance of the point (– 1, – 5, – 10) from the point of intersection of the line $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$. **6**

Q81. Find the distance of the point (2, 3, 4) from the line $\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$ measured parallel to the plane $3x + 2y + 2z - 5 = 0$. **6**

Q82. Find the coordinates of image of point (1, 3, 4) in the plane $2x - y + z + 3 = 0$. **6**

Q83. Find the image of the point having position vector $\hat{i} + 3\hat{j} + 4\hat{k}$ in the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$. **6**

Q84. Find the image of the point (3, – 2, 1) in the plane $3x - y + 4z = 2$. **6**

Q85. Find the coordinates of the foot of perpendicular and the perpendicular distance of point P(3, 2, 1) from the plane $2x - y + z + 1 = 0$. Find also image of the point in the plane. **6**

Q86. Find the length and foot of perpendicular from point P(7, 14, 5) to planes $2x + 4y - z = 2$. Also find the image of point P in the plane. **6**

Q87. Find the vector equation of the line passing through the point (1, 2, 3) and parallel to the planes $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$. **6**

Q88. Find the equation of the plane through the points (2, 1, – 1) and (–1, 3, 4) and perpendicular to the plane $x - 2y + 4z = 10$. Also, show that the plane thus obtained contains the line $\vec{r} = \hat{i} + 3\hat{j} + 4\hat{k} + \lambda(3\hat{i} - 2\hat{j} - 5\hat{k})$. **6**

Q89. Find the equation of plane passing through the line of intersection of planes $r \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $r \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ and parallel to x-axis. **6**

Q90. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}\cdot\left(\hat{i}+3\hat{j}\right)-6=0$ and $\vec{r}\cdot\left(3\hat{i}-\hat{j}-4\hat{k}\right)=0$, whose perpendicular distance from origin is unity. **6**

Q91. Find the equation of plane passing through the line of intersection of planes $2x+y-z=3$ and $5x-3y+4z+9=0$ and parallel to line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$. **6**

Q92. Find the vector equation of plane passing through the points $A(2, 2, -1)$, $B(3, 4, 2)$ and $C(7, 0, 6)$. Also, find the Cartesian equation of plane. **6**

Q93. Find the equation of plane passing through point (1, 1, – 1) and perpendicular to planes $x+2y+3z-7=0$ and $2x-3y+4z=0$. **6**

Q94. Find the vector and Cartesian equation of a plane containing the two lines $\vec{r}=\left(2\hat{i}+\hat{j}-3\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}+5\hat{k}\right)$ and $\vec{r}=\left(3\hat{i}+3\hat{j}+2\hat{k}\right)+\mu\left(3\hat{i}-2\hat{j}+5\hat{k}\right)$. Also, show that the line $\vec{r}=\left(2\hat{i}+5\hat{j}+2\hat{k}\right)+\lambda_1\left(3\hat{i}-2\hat{j}+5\hat{k}\right)$ lies in the plane. **6**

Q95. Find the equation of plane passing through point $P(1, 1, 1)$ and containing the line $\vec{r}=(-3\hat{i}+\hat{j}+5\hat{k})+\lambda(3\hat{i}-\hat{j}-5\hat{k})$. Also, show that plane contains the line $\vec{r}=(-\hat{i}+2\hat{j}+5\hat{k})+\mu(\hat{i}-2\hat{j}-5\hat{k})$. **6**

Q96. Find the equation of plane determined by the points $A(3, -1, 2)$, $B(5, 2, 4)$ and $C(-1, -1, 6)$. Also, find distance of point $P(6, 5, 9)$ from plane. **6**

Q97. Find the equation of plane passing through points (3, 4, 1) and (0, 1, 0) and parallel to line $\frac{x+3}{2}=\frac{y-3}{7}=\frac{z-2}{5}$. **6**

Q98. Find the equation of plane passing through the point A (1, 2, 1) and perpendicular to the line joining points P (1, 4, 2) and Q (2, 3, 5). Also, find distance of this plane from the line $\frac{x+3}{2}=\frac{y-5}{-1}=\frac{z-7}{-1}$. **6**

Q99. Find the equation of plane(s) passing through the intersection of planes $x+3y+6=0$ and $3x-y-4z=0$ and whose perpendicular distance from origin is unity. **6**

Q100 Find the foot of the perpendicular from the point (0, 2, 3) on the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$. Also, find the length of the perpendicular. **6**

Q101 *ABCD* is a quadrilateral such that $\overrightarrow{AB}=\vec{b}$, $\overrightarrow{AD}=\vec{d}$, $\overrightarrow{AC}=m\vec{b}+p\vec{d}$. Show that the area of the quadrilateral *ABCD is* **6**

$$\frac{1}{2}|m+p||\vec{b}\times\vec{d}|$$

Q102 Find the perpendicular distance of the point (2, 3, 4) from the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$. Also find coordinates of foot of perpendicular. **6**

Q103 Find the image of the point (1, 6, 3) on the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$. Also, write the equation of the line joining the given points and its image and find the length of segment joining the given point and its image. **6**

Q104 If $\vec{a}, \vec{b}, \vec{c}$ are such that $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $|\vec{c}| = 7$. Find angle between $\vec{a}$ and $\vec{b}$. **6**

Q105 Find a vector of magnitude 5 units, perpendicular to each of the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$, where $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}+3\hat{k}$. **6**

Q106 If with reference to a right handed system of mutually perpendicular unit vectors $\hat{i}, \hat{j}, \hat{k}$, we have $\vec{\alpha} = 3\hat{i} - \hat{j}$, and $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$. Express $\vec{\beta}$ in the form $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$ where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$. **6**

Q107 If $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = \hat{j}-\hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}\cdot\vec{c} = 3$. **6**

Q108 If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular vectors of equal magnitude, show that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$. Also find the angle. **6**

Q109 Find the values of c for which the vectors $\vec{a} = (c\log_2 x)\hat{i} - 6\hat{j} + 3\hat{k}$ and $\vec{b} = (\log_2 x)\hat{i} + 2\hat{j} + (2c\log_2 x)\hat{k}$ make an obtuse angle for any $x \in (0, \infty)$. **6**

Q110 Find the equation of the perpendicular drawn from the point (3, – 1, 11) to line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Also, find the coordinates of foot of perpendicular and the length of perpendicular. **6**

Q111 Find the length and foot of perpendicular drawn from the point (2, – 1, 5) to line **6**

$$\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}.$$

Q112 Find the equation of plane passing through the point (– 1, – 1, 2) and perpendicular to each planes $2x + 3y - 3z = 2$ and $5x - 4y + z = 6$. **6**