MATHEMATICS - XII

Probability , Value Base Questions -Solution

Name :.......................   Total Marks = 18

Time : 30 min   Date: 23/10/2017

S1. Here $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$

P (not A and not B) = $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

Now $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8} = \frac{5}{8}$$

P (not A and not B) = $1 - \frac{5}{8} = \frac{3}{8}$.

S2. Here $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\bar{A} \cup \bar{B}) = \frac{1}{4}$

Now $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B)$

$\therefore$ $\frac{1}{4} = 1 - P(A \cap B) \Rightarrow P(A \cap B) = 1 - \frac{1}{4} = \frac{3}{4}$

Now $P(A) \times P(B) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{24}$

$\therefore$ $P(A \cap B) \neq P(A) \times P(B)$

Thus A and B are not independent.

S3. Let X denote the number of hurdles knocked down by the player. Then, X follows binomial distribution with $n = 10$, $p = 1 - \frac{5}{6} = \frac{1}{6}$ and $q = \frac{5}{6}$.

$\therefore$ $P(X = r) = {}^{10}C_r \left(\frac{1}{6}\right)^r \left(\frac{5}{6}\right)^{10-r}$; $r = 0, 1, 2, \ldots, 10$

Required probability = $P(X < 2)$

$$= P(X = 0) + P(X = 1)$$

$$= \left(\frac{5}{6}\right)^{10} + {}^{10}C_1 \times \frac{1}{6} \times \left(\frac{5}{6}\right)^9$$

$$= \left(\frac{5}{6}\right)^9 \left\{\frac{5}{6} + \frac{10}{6}\right\} = \frac{5^{10}}{2 \times 6^9}$$

S4. Let X denote the number of defective eggs in a sample of 10 eggs drawn successively with replacement. Then, X follows binomial distribution with parameters n = 10, $p = \frac{10}{100} = \frac{1}{10}$ and $q = \frac{9}{10}$.

$$\therefore \qquad P(X = r) = {}^{10}C_r \left(\frac{1}{10}\right)^r \left(\frac{9}{10}\right)^{10-r}, \; r = 0, 1, 2, \ldots.., 10$$

Required probability = $P(X \geq 1) = 1 - P(X = 0)$

$$= 1 - {}^{10}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{10} = 1 - \frac{9^{10}}{10^{10}}$$

S5. Here,

$$n = 20, \quad p = \frac{1}{2}, \quad q = \frac{1}{2}$$

$$\therefore \qquad P(X = r) = {}^{20}C_r \left(\frac{1}{2}\right)^{20}$$

Required probability = $P(X \geq 12) = \sum_{r=12}^{20} {}^{20}C_r \left(\frac{1}{2}\right)^{20}$

S6. Let X denote the number of defective bulbs in a sample of 4 bulbs drawn successively with replacement. Then, X follow binomial distribution with parameters n = 4, $p = \frac{6}{30} = \frac{1}{5}$ and $q = 1 - \frac{1}{5} = \frac{4}{5}$ such that

$$\text{P}(X = r) = {}^{4}C_r \left(\frac{1}{5}\right)^r \left(\frac{4}{5}\right)^{4-r}; \quad r = 0, 1, 2, 3, 4.$$

The probability distribution of X is

X:	0	1	2	3	4
$P(X)$:	$\left(\frac{4}{5}\right)^4$	$\left(\frac{4}{5}\right)^4$	$6 \times \frac{1}{25} \times \frac{16}{25} = \frac{96}{625}$	$4 \times \frac{1}{125} \times \frac{4}{5} = \frac{16}{625}$	$\left(\frac{1}{5}\right)^4$

S7. The distribution of the 'number of successes' is a binomial distribution with n = 20 and,

$$p = \text{Probability of getting a number greater than 4} = \frac{2}{6} = \frac{1}{3}$$

$$\therefore \qquad q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$$

Now $\qquad$ Mean = np $\quad$ and $\quad$ Variance = npq

$\Rightarrow$ Mean $= 20 \times \frac{1}{3} = 6.66$ and Variance $= 20 \times \frac{1}{3} \times \frac{2}{3} = 4.44$

Hence, Mean = 6.66 and Variance = 4.44.

S8. Let X be a binomial variate with parameters n and p. Then,
We have,

p = Probability of getting a total greater than 4 in a single throw of a pair of dice.

$$\Rightarrow \quad p = 1 - \frac{6}{36} = \frac{5}{6}$$

$$\Rightarrow \quad q = 1 - p = 1 - \frac{5}{6} = \frac{1}{6}$$

$$\therefore \quad \text{Mean} = np = \frac{5}{6} \times 12 = 10$$

and, $$\text{Variance} = npq = 12 \times \frac{5}{6} \times \frac{1}{6} = \frac{5}{3}$$

S9. Let n denote the number of throws required to get a six and X denote the amount won/lost.

The man may get a six in the very first throw of the die or in 2^{nd} throw or in the third throw (as he has decided to throw a die at most thrice).

Thus, we have the following probability distribution for X.

Number of throws (n) :	1	2	3	3
Amount won/lost (X) :	1	0	– 1	– 3
Probability ($P(X)$) :	$\frac{1}{6}$	$\frac{5}{6} \times \frac{1}{6}$	$\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$	$\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$

$$\therefore \quad E(X) = 1 \times \frac{1}{6} + 0 \times \frac{5}{36} + (-1) \times \frac{25}{216} + (-3)\frac{125}{216} = -\frac{364}{216}$$