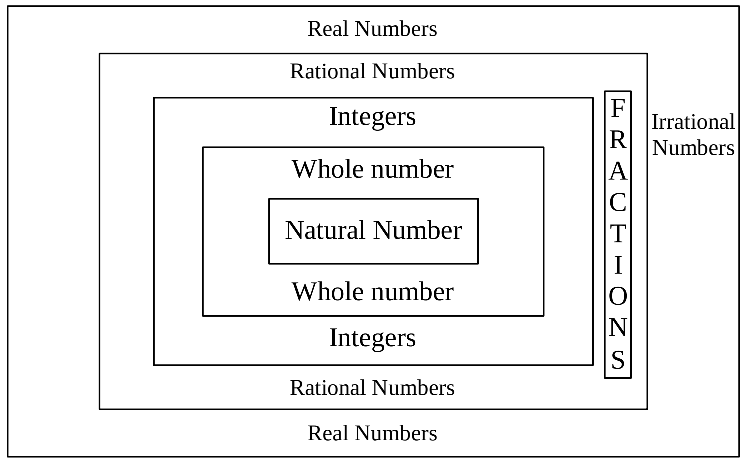
1 REAL NUMBERS

1.1 INTRODUCTION

We studied about real numbers



Natural numbers: The counting numbers 1,2,3..... are called natural numbers. It is denoted by N.

N = {1,2.3..........}

Whole numbers: In the set of natural number if we include the number 0, the resulting set is known as the set of whole numbers.

It is represented by W.

W = {0,1,2.......}

Integers: Natural numbers along with 0 and their negatives are called integers and the set of integers is denoted by I

I = {.........–4, –3, –2, –1, 0, 1,2,3..........}

Rational numbers: A rational number is a number which can be expressed in the form of p/q, where p and q are integers and q is not zero.

Irrational number: A number is called irrational if it can not be written in the form of p/q, where p and q are integers and $q \neq 0$

The system R of real numbers includes rational as well irrational numbers.

In this chapter we will begin with a brief recall of divisibility of integers as well state some important properties of integers.

1.2 DIVISIBILITY

A non zero integer 'a' is said to divide an integer 'b' if there exists an integer 'c' such that b = ac

The integer 'a' is called the dividend, integer 'b' is known as the divisor and integer 'c' is known as the quotient. For example, 3 divides 36 because there is an integer 12 such that 36 = 3 × 12. However, 3 does not divide 35 because there do not exist an integer 'c' such that 35 = 3 × c. In other words, 35=3×c is not true for any integer 0.

Note: If a non zero integer 'a' divides an integer 'b', then we write a | b. This is read as "a divides b". When a | b, we say that 'b is divisible by a' or 'a is a factor of b' or 'b is a multiple of a' or 'a is divisor of b'.

Some Properties of Divisibility

(i) ±1 divides every non-zero integer.

(ii) 0 does not divide any integer.

(iii) If a is a non zero integer and b is any integer, then $a \mid b \Rightarrow a \mid -b, -a \mid b$ and $-a \mid -b$.

(iv) If a and b are non-zero integers, then

$a \mid b$ and $b \mid a \quad \Rightarrow a = \pm b$

(v) If a is a non-zero integer and b,c are any two integers, then

$$a \mid b \text{ and } a \mid c \Rightarrow \begin{cases} a \mid b \pm c \\ a \mid bc \\ a \mid bx \text{ for any integer } x \end{cases}$$

(vi) If a and c are non zero integers and b,d are any two integers, then

(a) $a \mid b$ and $c \mid d \Rightarrow ac \mid bd$

(b) $ac \mid bc \Rightarrow a \mid b$

1.3 EUCLID'S DIVISION LEMMA

(a) Euclid's division Lemma: Let *a* and *b* be any two positive integers. Then, there exist unique *q* and *r* such that

$$a = bq + r, 0 \le r < b$$

If b | a, then r = 0, otherwise, r satisfies the strict inequality.

$$0 < r < b$$

1.2.1 Highest Common Factor (HCF)

HCF of two or more numbers is the largest number that divides all the given numbers completely

It is also called the Greatest Common Divisor (GCD).

1.2.2 Lowest or least Common Factor (LCM)

The LCM of two or more numbers is the smallest number which is multiple of each of the numbers or in other words the LCM of two or more numbers is the smallest number which is divisible by all the given numbers.

Properties of HCF and LCM of given numbers

(i) The HCF of given numbers is *not greater than* any of two numbers.

(ii) The LCM of given numbers is *not less than* any of given numbers.

(iii) The HCF of a given number is always factor of their LCM

(iv) The HCF of two coprime numbers is 1.

(v) The LCM of two or more coprime numbers is equal to their product.

(vi) If a and b are two numbers then

$$a \times b = \text{HCF}(a, b) \times \text{LCM}(a, b)$$

(vii) If a, b, c are positive integers

$$LCM(a,b,c) = \frac{a,b,c\ HCF(a,b,c)}{HCF(a,b)\ HCF(b,c)\ HCF(a,c)}$$

$$HCF(a,b,c) = \frac{a,b,c\ LCM(a,b,c)}{LCM(a,b)\ LCM(b,c)\ LCM(a,c)}$$

Coprime numbers: Two numbers are said to be coprime if they are relatively prime i.e. their GCD is 1. For Example 5 and 7 are coprime because 5 and 7 are coprime because 5 and 7 are relatively prime.

Prime factorization method to find HCF

Step–1: Find prime factorisation of each of the given number

Step–2: Identify common prime factors

Step–3: Find the product of all the common prime factors, using each common prime factor the least number of times it appears in the prime factorisation of any of the given numbers. The product so obtained is the required HCF.

Prime factorization method to find LCM

Step–1: Write the prime factorisation of each of the given numbers.

Step–2: Find the product of all different prime factors of the numbers using each common prime factor the highest number of times it appears in the prime factorisation of any of the numbers. The product so obtained is the required LCM of the given number.

Composite numbers:

A composite number is a positive integer which has a positive divisor other than one or itself. In other words, if $n > 0$ is an integer and there are integers $1 < a, b < n$ such that $n = a \times b$, then n is composite. By definition, every integer greater than one is either a prime number or a composite number. For example, the integer 14 is a composite number because it can be factored as 2×7. Likewise, the integers 2 and 3 are not composite numbers because each of them can only be divided by one and itself.

1.3.3 Euclid's Division Alogrithm

Algorithm: An algorithm is a series of well defined steps which provide a procedure of calculation repeated successively on the results of earlier steps till the desired result is obtained.

Euclid's division algorithm is an algorithm to compute the highest common factor (HCF) of two given positive integers.

Some Important Results

(i) If b is a factor of a, then HCF of (a,b) = b which is simply written as (a,b) = b.

(ii) If a = q.b+r, r < b then HCF of (a,b) = HCF of (b, r) or (a,b) = (b,r)

(iii) If (a,b) = 1 and bc is divisible by a, then c is divisible by a. This is known as Gauss's Theorem.

(iv) If a and b are primes and a | bc ⇒ a | c

(v) The HCF(d) of two positive integers a and b can be expresed as a linear combination of a and b i.e. d = *x*a + yb for some integers *x* and y. Also this representation is not unique.

1.4 THE FUNDAMENTAL THEOREM OF ARITHMETIC

1.4.1 Theorem (Fundamental Theorem of Arithmetic)

Every composite number can be expressed (factorised) as a product of primes and this factorisation is unique, apart from the order in which the prime factors occurs.

The prime factorisation of a natural number is unique except the order of its factors.

Let x be a composite number we factorise it as $x = p_1 \times p_2 \times p_3 ....p_n$, where $p_1, p_2 ...p_n$ are primes and written in ascending order i.e. $p_1 \le p_2 \le p_3 ...p_n$. If we combine we will get powers of primes.

1.4.2 Theorem

Let p be a prime number and a be positive integer. If p divides a^2 then p also divides a

1.5 DETERMINING THE NATURE OF THE DECIMAL EXPANSION OF RATIONAL NUMBERS

We have studied that the decimal expansion of a rational number is either terminating or non terminating repeating (or recurring) without knowing when it is terminating and when it is non-terminating repeating. Here in this section we will explore exactly when the decimal expansion of a rational number is terminating and when it is non terminating repeating.

1.5.1 Theorem

Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form p/q where p and q are coprimes and the prime factorisation of q is of the form $2^m \times 5^n$ where m and n are non negative integers.

1.5.2 Theorem

Let x = p/q be a rational number such that the prime factorization of q is the form of $2^m \times 5^n$ where m and n are non-negative integers. Then x has a decimal expansion which terminates.

1.5.3 Theorem

Let x = p/q be a rational number such that prime factorization of q ***is not*** of the form $2^m \times 5^n$ where m and n are non negative integers. Then x has a decimal expansion which is non-terminating repeating.

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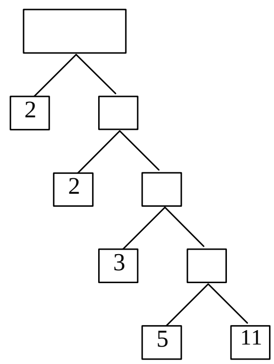
EXERCISE-I

1. Check whether 3^n can end with the digit 0 for any $n \in N$.
2. Find HCF and LCM of 18 and 24 by prime factorisation
3. Using prime factorization method find HCF. of 144, 198
4. Using prime factorisation method find LCM of 12, 15, 20, 27.
5. Given that HCF (1152, 1664) = 128 find LCM of (1152, 1664)
6. Find the missing number in the following factorisation sequence.



7. Find the HCF of 65 and 117 and express it in the form 65m + 117 n.

EXERCISE-II

1. Find HCF of 18,24,36, and 48.
2. A heap of coconuts is divided into groups of 2, 3 and 5 and each time no coconut is left over find the least number of coconut in the heap.
3. If the HCF of 210 and 55 is expressible in the form of $210 \times 5 + 55y$. Find y
4. Find the largest number that divides 2053 and 967 leaves remainder of 5 and 7 respectively.
5. Two tankers contains 850 litres and 680 litres of petrol respectively. Find the maximum capacity of a container which can measure the petrol of either tank in exact number of times.
6. A mason has to fit a bathroom with square marble tiles of the largest possible size. The size of the bathroom is 10 ft by 8 ft. What would be the size in inches of the tile required that has to be cut and how much such tiles are required?
7. The length, breadth and height of a room are 8 m 25 cm, 6 m 75 cm and 4m 50 cm respectively. Determine the longest rod which can measure the three dimensions of the room exactly.

EXERCISE-III

SECTION-A

- **Assertion & Reason**

Instructions: In the following questions an Assertion (A) is given followed by a Reason (R). Mark your responses from the following options.

(A) Both Assertion and Reason are true and Reason is the correct explanation of 'Assertion'.

(B) Both Assertion and Reason are true and Reason is not the correct explanation of 'Assertion'.

(C) Assertion is true but Reason is false

(D) Assertion is false but Reason is true

1. **Assertion:** For any integers a and b with $b \neq 0$ there exist unique integers q and r, such that a = bq + r, $0 \leq r < |b|$

Reason: An integer a which is not exactly divisible by 3 can be written in one of the forms a = 3n + 1 or a = 3n + 2 where n is an integer.

2. **Assertion:** 9/4608 is a rational number.

Reason: All terminating or non terminating recurring numbers are rational numbers.

SECTION-B

- **Match the following (one to one)**

Column-I and **column-II** contains **four** entries each. Entries of column-I are to be matched with some entries of column-II. Only One entries of column-I may have the matching with the some entries of column-II and one entry of column-II Only one matching with entries of column-I

1.

Column I	**Column II**
(A) –10	(P) Natural number
(B) π	(Q) Integer but not a natural number
(C) 3	(R) Rational number but not an integer
(D) 5/2	(S) Irrational

EXERCISE-IV

SECTION-A

- **Multiple choice question with one correct answer**

1. If x, y and z are real numbers such that $x < y$ (x, y > 0) and z < 0, then the statement which is true is

(A) $xz < yz$ (B) $\left(\frac{x}{z}\right) < \left(\frac{y}{z}\right)$ (C) $\left(\frac{z}{x}\right) > \left(\frac{z}{y}\right)$ (D) $xy > yz$

2. Which of the following is a pair of coprimes?
(A) (14, 35) (B) (18,25) (C) {31,93} (D) (32,62)
3. HCF of $2^3 \times 3^2 \times 5$, $2^3 \times 3^3 \times 5^2$ and $2^2 \times 3 \times 5^3 \times 7$ is
(A) 30 (B) 48 (C) 60 (D) 105
4. The product of two digit number is 2160 and their GCM is 12. The numbers are
(A) 72, 30 (B) 36, 60 (C) 96, 25 (D) None
5. LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 280, then the other is
(A) 40 (B) 60 (C) 80 (D) 100
6. The ratio of two numbers is 3 : 4 and their HCF is 4. Then LCM is
(A) 12 (B) 16 (C) 24 (D) 48
7. LCM of 2 prime numbers x and y (x > y) is 161. The value of 3y–x is
(A) –2 (B) –1 (C) 1 (D) 2
8. The HCF of $(4a^2b^3–9b)$ and $(2a^2b^2–ab–3)$ is
(A) (2a–3) (B) (2ab–3) (C) (2b–3a) (D) None of these
9. What is the greatest number which shall divide 305 and 629 and leave a remainder 8 in each case?
(A) 24 (B) 36 (C) 27 (D) 35
10. Given x and n are integers $(15n^2 + 6n^2 + 5n + x)/n$ is not an integer for what condition?
(A) n is potive
(B) x is divisible by n
(C) x is not divisible by n
(D) both (A)&(C)

SECTION-B

- **Multiple choice question with one or more than one correct answers**

1. The set of even integers E = {.........–4, –2, 0, 2, 4, 6} is closed under operation
(A) Addition (B) Subtraction (C) Multipication (D) Division
2. If n is a natural number, then $\sqrt{n}$ can be
(A) a natural number
(B) always an rational number
(C) an irrational number
(D) always a natural number
3. a and b are prime numbers which of the following is/are true?
(A) a^2 has 3 positive integral factors
(B) ab has 4 prositive integral factors
(C) a^3 has positive integral factors
(D) a^2b^2 has 4 positive integral factors
4. Which of the following statements for natural numbers a, b and c is/are true
(A) If a is divisible by b and b is divisible by c, then a must be divisible by c.
(B) If a is a factor of both b and c, then a must be a factor of b+c
(C) If a is a factor of both b and c then a must be a factor of b–c.
(D) If a is a factor of b and b anc c are coprime, then a,c must also be coprimers.
5. Which of the following rational numbers have terminating decimal expansion
(A) 64/455 (B) 29/343 (C) 13/325 (D) 1/308
6. Which one of the following is correct?

The number $\sqrt{14+6\sqrt{5}}+\sqrt{14-6\sqrt{5}}$

(A) is a rational numbers (B) is not a rational number (C) simplifies to 5 (D) simplifies to 6

SECTION-C

- **Match the following (one to many)**

Column-I and **column-II** contains **four** entries each. Entries of column-I are to be matched with some entries of column-II. One or more than one entries of column-I may have the matching with the some entries of column-II and one entry of column-II may have one or more than one matching with entries of column-I

1.

Column I	Column II
(A) If two irrational number is added then sum may be	(P) Irrational number
(B) Division of a natural number by another natural number gives	(Q) Rational number
(C) Natural number	(R) Neither rational nor irrational
(D) Every real number	(S) None of these

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Answers

EXERCISE-I

2. HCF = 6 and LCM = 72 3. HCF = 18 4. LCM = 540
5. 14976 6. Going upwards 55, 165, 330, 660 7. 13

EXERCISE-II

1. HCF = 6 (Hint: HCF of 18 and 36 = 18; HCF of 24 and 48 = 24; HCF of 18, 24 = 6)
2. 30 3. $y = -19$
4. 64 (Hint: 2053 – 5 = 2048 $\Rightarrow$ 967 – 7 = 960)
5. 170 litres
6. 24 inch, 20 tiles 7. 75 cm

EXERCISE-III

Section-A

1. (B) 2. (A)

Section-B

1. (A)-(Q), (B)-(S), (C)-(P), (D)-(R)

EXERCISE-IV

Section-A

1. (D)	2. (B)	3. (C)	4. (B)
5. (C)	6. (D)	7. (A)	8. (B)
9. (C)	10. (C)		

Section-B

1. (ABC)	2. (AC)	3. (ABC)	4. (ABCD)
5. (BC)	6. (AD)		

Section-C

1. (A)-(P,Q), (B)-(Q), (C)-(Q), (D)-(P,Q)

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