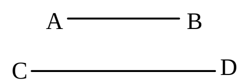
4 SIMILAR TRIANGLES

4.1 INTRODUCTION

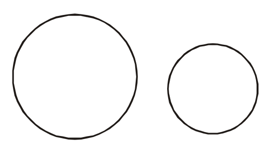
In previous classes, we have learnt about the congruency of two geometric figure. In this chapter we shall learn about these geometric figures. Which have the same but not necessary have the same size. These kind of geometric figures are known as simmilar figures. So the congruent figures are always similar figures but similar figures need not be congruence figures.

(i) Two line segment are similar



(The two line segment are congruent if they have the same length)

(ii) Two circles are similar _____________. (The two circles are congruent. If they have the same radius)



4.2 SIMILAR TRIANGLE

4.2.1 Similar Polygons

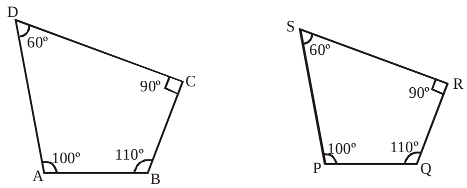
Two polygons of the same number of sides are said to be similar. If

(i) Their corresponding angles are equal

(ii) Their corresponding sides are in the same ratio

Illustration 1

If two polygons ABCD and PQRS are similar then



Solution

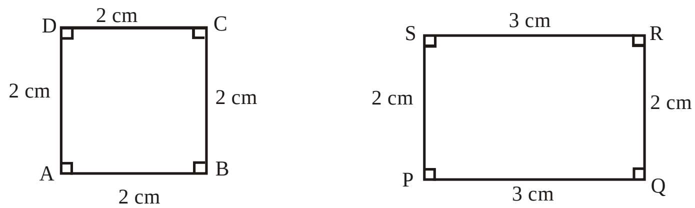
By the definition

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CD}{RS} = \frac{DA}{SP} = 2$$

So corresponding sides are proportional.

Therefor quadrilateral ABCD and PQRS are similar

Illustration 2



Solution

Clarly A square ABCD and rectengle PQRS are equiangular.

But corresponding sides of square ABCD and rectangle PQRS are not proportional.

Therefor square ABCD and rectangle PQRS are not similar.

Remark: If one polygon is similar to a second polygon and the second polygon is similar to a third polygon, then the first polygon is similar to the third polygon.

4.3 SIMILARITY OF TRIANGLES

Two triangles are said to be similar if

(i) Their corresponding angles are equal (or triangles are equiangular)

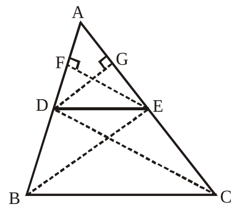
(ii) Their corresponding sides are in the same ratio (or proportional)

Remark: equiangular triangles means that the corresponding angles of the triangles are equal.

Theorem 1: (Basic proportionality theorem or thales theorem)

If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.

$$\frac{AD}{DB} = \frac{AE}{EC}$$



4.3.1 Carollary

If in a ΔABC, a line DE || BC, intersects AB in D and AC in E then

(i) $\frac{AB}{AD} = \frac{AC}{AE}$ (ii) $\frac{AD}{DB} = \frac{AC}{EC}$

(ii) from the basic proportionality theorem

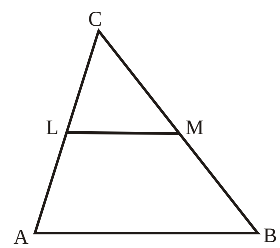
Theorem–2: Converse of basic proportionality Theorem

If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

Illustration 3

LM || AB. If AL = x – 3, AC = 2x, BM = x – 2, BC = 2x + 3

find the value of x?



Solution

In Δ ABC we have

LM || AB

$$\frac{AL}{LC} = \frac{BM}{MC} \Rightarrow \frac{AL}{AC - AL} = \frac{BM}{BC - BM}$$

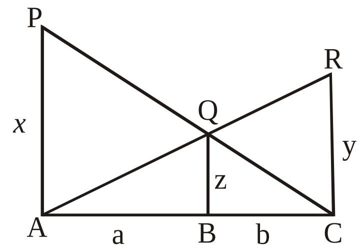
$$\frac{x-3}{2x-(x-3)} = \frac{x-2}{(2x+3)-(x-2)}$$

$$\Rightarrow \frac{x-3}{x+3} = \frac{x-2}{x+5} \qquad \Rightarrow (x-3)(x+5) = (x-2)(x+3)$$

$$\Rightarrow x^2 + 2x - 15 = x^2 + x - 6 \qquad \Rightarrow x = 9$$

Illustration 4

In the given figure PA, QB and RC each is perpendicular to AC such that PA = x, RC = y, QB = z, AB = a and BC = b. Prove that $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$.



Solution

PA ⊥ AC and QB ⊥ AC $\Rightarrow$ QB || PA

Thus in Δ PAC, QB || PA

so Δ QBC ~ Δ PAC

$\therefore \quad \frac{QB}{PA}=\frac{BC}{AC} \qquad \Rightarrow \frac{z}{x}=\frac{b}{a+b}$ ..... (i) (by the property of similar triangle)

In Δ RAC, QB || RC, so Δ QBC ~ Δ RAC

$\therefore \quad \frac{QB}{RC}=\frac{AB}{AC} \qquad \Rightarrow \frac{z}{y}=\frac{b}{a+b}$ ..... (ii) (by the property of similar triangle)

Now from (i) and (ii) we get

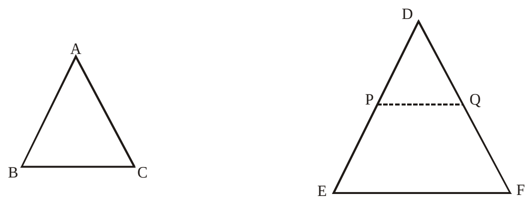
$$\frac{z}{x}+\frac{z}{y}=\left(\frac{b}{a+b}+\frac{a}{a+b}\right)=1$$

$$\frac{z}{x}+\frac{z}{y}=1$$

$$\left(\frac{1}{x}+\frac{1}{y}=\frac{1}{z}\right)$$

THEOREM–3: (AAA - SIMILARITY)

If in two triangle. The corresponding angles are equal, then their corresponding sides are proportional and hence the triangles are similar.



4.4.1 Corollary (AA Similarity)

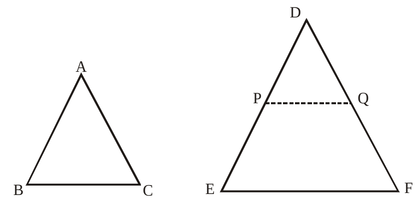
If two angles of one triangle are respectively equal to two angles of another triangle than the two triangles are similar.

Theorem–4: (SSS-Similarity)

If the corresponding sides of two triangles are proportional then their corresponding angles are equal and hence the two triangles are similar.

Given :

Δ ABC and Δ DEF & $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$



Theorem–5: (SAS-similarity)

If one angles of a triangles is equal to one angle of the other triangle and the sides including these angles are proportional then two triangles are similar.

ΔABC and ΔDEF

$\angle A = \angle D$ & $\dfrac{AB}{DE} = \dfrac{AC}{DF}$

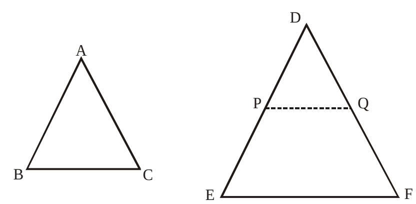


Illustration 5

Prove that the ratio of the perimeters of two similar triangles is the same as the ratio of their corresponding sides.

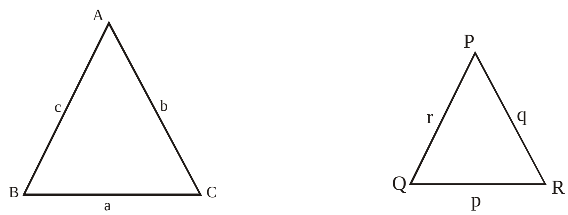
Solution

Given: Δ ABC and Δ PQR

BC = a, CA = b, AB = c

and QR = p, RP = q, PQ = r

Also Δ ABC ~ Δ PQR



To prove: $\dfrac{a}{p} = \dfrac{b}{q} = \dfrac{c}{r} = \dfrac{a+b+c}{p+q+r}$

Proof: Since Δ ABC and Δ PQR are similar, there for their corresponding sides are proportional

$\therefore \quad \dfrac{a}{p} = \dfrac{b}{q} = \dfrac{c}{r} = k$ (say)

a = kp, b = kq, c = kr

$$\therefore \quad \frac{\text{perimeter of } \Delta ABC}{\text{perimeter of } \Delta PQR} = \frac{a+b+c}{p+q+r} = \frac{kp+kq+kr}{p+q+r}$$

$$= \frac{k(\cancel{p}+\cancel{q}+\cancel{r})}{\cancel{p}+\cancel{q}+\cancel{r}} = k \qquad \text{..... (ii)}$$

from (i) and (ii) we get

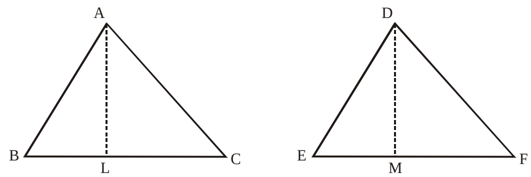
$$\frac{a}{p} = \frac{b}{q} = \frac{c}{r} = \frac{a+b+c}{p+q+r} = \frac{\text{perimeter of } \Delta ABC}{\text{perimeter of } \Delta PQR}$$

4.4 AREAS OF SIMILAR TRIANGLES

Theorem–6: The ratio of the area of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Given: Δ ABC ~ Δ DEF

To Prove : $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2}$



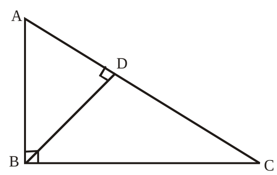
4.5 PYTHAGORAS THEOREM

Theorem–7: In a right triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Given : In Δ ABC

$\angle ABC = 90^\circ$

To prove: $AC^2 = AB^2 + BC^2$



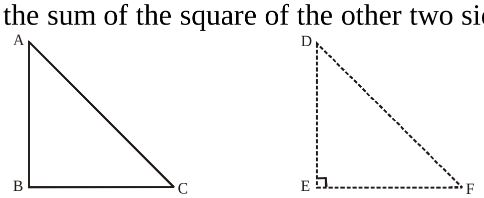
Theorem–8: Converse of Pythagoras theorem

In a triangle , if the square of one side is equal to the sum of the square of the other two sides then the angle opposite to the first side is right angle.

In Δ ABC

$AC^2 = AB^2 + BC^2$

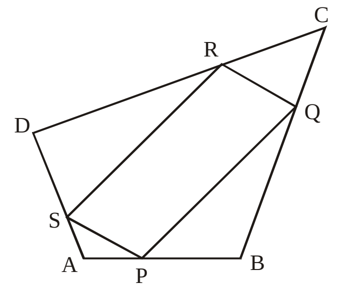
$\angle B = 90^\circ$



Solved Examples

Example 1

In the adjoining figure. ABCD is a quadrilateral and P, Q, R, S are the points of trisection of the sides AB, BC, CD and DA respectively. Prove that PQRS is a parallelogram.



Solution

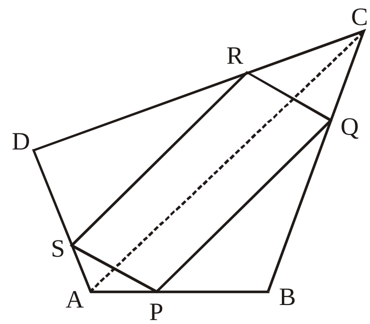
Here, ABCD is a quadrilateral. Since R and S are points of trisection of sides CD and DA respectively.

$\therefore$ CD = 3CR or CR + DR = 3CR

or DR = 2CR or $\frac{DR}{RC} = \frac{2}{1}$ and

AD = 3AS or AS + SD = 2AS

or DS = 2AS or $\frac{DS}{SA} = \frac{2}{1}$



$\Rightarrow \frac{DS}{SA} = \frac{DR}{RC}$

$\therefore$ By converse of basic proportionality theorem,

In Δ DAC, $\frac{DS}{SA} = \frac{DR}{RC}$ $\Rightarrow$ SR || AC

Similarly, PQ || AC

$\therefore$ SR || AC and PQ || AC $\Rightarrow$ SR || PQ

$\Rightarrow$ Similarly one can prove that PS || QR

Hence, PQRS is a parallelogram.

Example 2

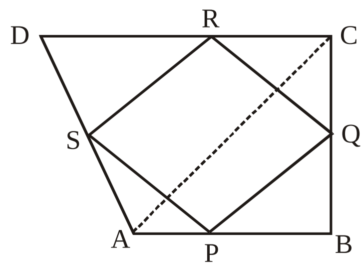
Prove that the line segments joining the mid-points of the adjacent sides of a quadrilateral form a parallelogram.

Solution

Let P, Q, R and S respectively be the mid-points of the sides AB, BC, CD and DA of the quadrilateral ABCD.

Join PQ, QR, RS and SP.

Also, join AC,



Since S and R are the mid-points of DA and DC respectively

$\therefore$ DS = SA and DR = RC

$\Rightarrow \frac{DS}{SA} = 1$ and $\frac{DR}{RC} = 1$

$\Rightarrow \frac{DS}{SA} = \frac{DR}{RC}$

$\therefore$ In Δ DAC, $\frac{DS}{SA} = \frac{DR}{RC}$ $\Rightarrow$ SR || AC ..... (i) (By converse of basic proportionality theorem)

Since Q and P are the mid-points of BC and BA respectively.

$\therefore$ BQ = QC and BP = PA

$\Rightarrow \frac{BQ}{QC} = 1$ and $\frac{BP}{PA} = 1$

$\Rightarrow \frac{BQ}{QC} = \frac{BP}{PA}$

$\therefore$ In Δ BCA, $\frac{BQ}{QC} = \frac{BP}{PA}$

$\Rightarrow$ QP || CA or PQ || AC ..... (ii) (By converse of basic proportionality theorem)

From (i) and (ii), we have

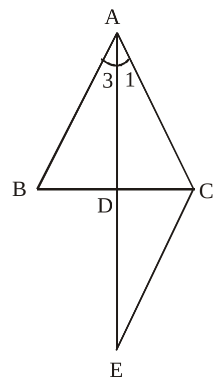
PQ || SR

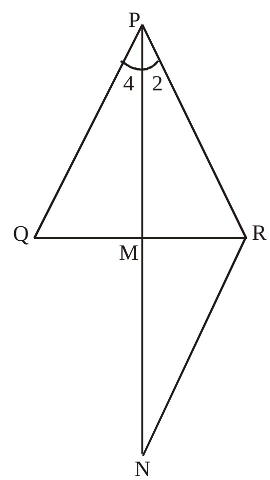
Similarly, PS || QR

Hence, PQRS is a parallelogram.

Example 3

Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that Δ ABC ~ ΔPQR.





Solution

Since sides AB and AC and median AD of a triangle ABC are respectively proportinal to sides PQ and PR and median PM of another triangle PQR.

$\therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

Now, produce AD to E and PM to N such that AD = DE and PM = MN.

Join EC and NR

In triangles ADB and EDC,

AD = DE (construction)

BD = CD ($\because$ AD is median)

$\angle$ADB = $\angle$EDC (Vertically opposite angles)

$\therefore$ By S.A.S. congruency criteria,

Δ ADB $\cong$ Δ EDC

$\Rightarrow$ AB = EC ($\because$ corresponding parts of congruent triangles are equal)

Now, in triangles PMQ and NMR,

PM = MN (construction)

QM = RM ($\because$ PM is median)

$\angle$PMQ = $\angle$NMR (vertically opposite angles)

∴ By S.A.S. congruency criteria,

$\Delta PMQ \cong \Delta NMR$

$\Rightarrow$ PQ = NR (∵ corresponding parts of congruent triangles are equal)

Now, $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

$\Rightarrow \frac{EC}{NR} = \frac{AC}{PR} = \frac{AD}{PM}$ [∵ AB = EC and PQ = NR (proved above)]

$\Rightarrow \frac{EC}{NR} = \frac{AC}{PR} = \frac{2AD}{2PM} \Rightarrow \frac{EC}{NR} = \frac{AC}{PR} = \frac{AE}{PN}$

$\Rightarrow \Delta AEC \sim \Delta PNR$ (By using S.S.S. similarity criteria)

$\Rightarrow \angle 1 = \angle 2$ (∵ Similar triangles are equiangular)

Similarly, we can prove that $\angle 3 = \angle 4$

$\therefore \angle 1 + \angle 3 = \angle 2 + \angle 4$

$\Rightarrow \angle BAC = \angle QPR$

Now, in triangles ABC and PQR,

$\angle BAC = \angle QPR$ (prove above)

$\frac{AB}{PQ} = \frac{AC}{PR}$ (given)

∴ By S.A.S. similarity criteria,

$\Delta ABC \sim \Delta PQR$

Hence proved.

Example 4

Two poles of heights a metres and b metres are p metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{ab}{a+b}$ ***metres.***

Solution

Let AL and BM represent the two poles of heights a metres and b metres repectively.

Since the poles are p metres apart.

$\therefore$ LM = p metres

Let O be the point of intersection of the lines AM and BL

From O, draw On perpendicular on LM

Let ON = h metres and LN = x metres

$\therefore$ Nm = LM – LN = (p – x) metres

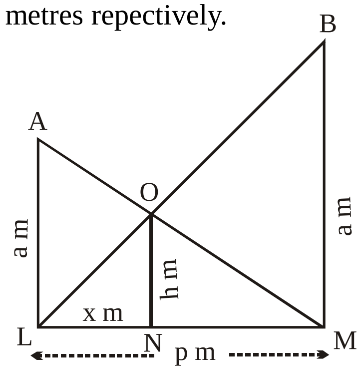
In triangles LMB and LNO,

$\angle LMB = \angle LNO$ (Each = 90º)

$\angle L = \angle L$ (common)

∴ By A.A. similarity criteria

$\Delta LMB \sim \Delta LNO$



$\Rightarrow \dfrac{LM}{LN} = \dfrac{MB}{NO} \qquad \Rightarrow \dfrac{p}{x} = \dfrac{b}{h} \qquad \Rightarrow x = \dfrac{ph}{b}$ ..... (i)

In triangles MLA and MNO,

$\angle MLA = \angle MNO$ (each 90º)

$\angle LMA = \angle NMO$ (common)

$\therefore$ By A.A. similarity criteria,

Δ MLA ~ Δ MNO

$\Rightarrow \dfrac{ML}{MN} = \dfrac{LA}{NO} \qquad \Rightarrow \dfrac{p}{p-x} = \dfrac{a}{h} \qquad \Rightarrow p - x = \dfrac{ph}{a} \quad \Rightarrow x = p - \dfrac{ph}{a}$ ..... (ii)

From (i) and (ii), we have

$$\frac{ph}{b} = p - \frac{ph}{a}$$

$\Rightarrow \dfrac{ph}{a} + \dfrac{ph}{b} = p \qquad \Rightarrow ph\left(\dfrac{1}{a} + \dfrac{1}{b}\right) = p \qquad \Rightarrow h\left(\dfrac{b+a}{ab}\right) = 1$

$\Rightarrow \quad h = \dfrac{ab}{a+b}$

Hence, the required height is $\dfrac{ab}{a+b}$ metres.

Example 5

In an equilateral triangle ABC, D is a point on side BC such that BD = 1/3 BC. Prove that $9AD^2 = 7AB^2$.

Solution

In equilateral Δ ABC, D is a point on side BC such that BD = $\dfrac{1}{3}$BC.

From A, draw AE $\perp$ BC.

Also, join AD

Now, in right angled triangles AEB and AEC,

$\angle B = \angle C$ (each 60º)

$\angle AEB = \angle AEC$ (each 90º)

$\therefore$ By A.A. similarity criteria,

Δ AEB ~ Δ AEC

$\Rightarrow \dfrac{AB}{AC} = \dfrac{AE}{AE} = \dfrac{BE}{CE}$ ($\because$ corresponding sides of similar triangles are proportional)

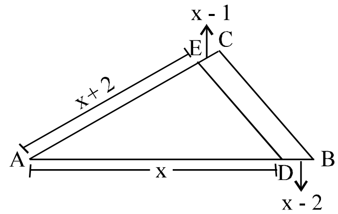
$\Rightarrow \dfrac{AB}{AC} = 1 = \dfrac{BE}{CE}$

$\Rightarrow \dfrac{BE}{CE} = 1$

*****

EXERCISE-I

1. In the given figure, DE || BC. If AD = x, DB = x – 2, AE = x + 2 and EC = x – 1, find the value of x.



2. ABCD is a parallelogram, P is a point on side BC and DP, when produced meets AB produced at L. Prove that (i) $\frac{DP}{PL} = \frac{DC}{BL}$, (ii) $\frac{DL}{DP} = \frac{AL}{DC}$.

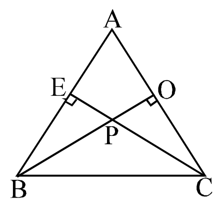
3. In a ΔABC, AD is the bisector of $\angle$A, meeting side BC at D. If AC = 4.2cm, DC = 6cm, BC = 10cm, find AB.

4. In ΔABC, if AD is the bisector of $\angle$A, Prove that $\frac{Area(\Delta ABD)}{Area(\Delta ACD)} = \frac{AB}{AC}$.

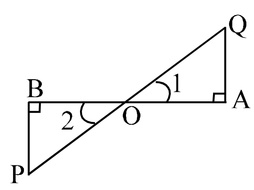
5. In ΔABC, D is the mid point of BC and ED is the bisector of $\angle$ADB and EF is drawn parallel to BC cutting AC in F. Prove that $\angle$EDF is a right angle.

6. Two triangles ΔBAC and ΔBDC, right angled at A and D respectively, are drawn on the same base BC and on the same side of BC. If AC and DB intersect at P, Prove that AP × PC = DP × PB.

7. In the given figure, considering the tiangles BEP and CPO, prove that BP × PD = EP × PC.



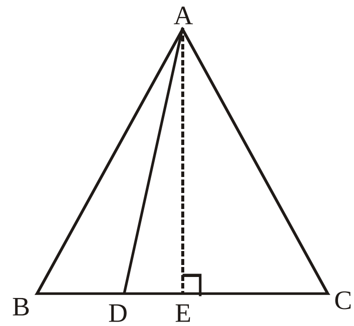
8. In the given figure PB and QA perpendicular to segment AB. If PO = 5cm, QO = 7cm and area ΔPOB = 150cm^2, find the area of ΔQOA.



9. In the given figure, ΔABC and ΔDBC are two triangles on the same base BC. If AD intersects BC at O, prove

that $\frac{area(\Delta ABC)}{area(\Delta DBC)} = \frac{AO}{DO}$.

$\Rightarrow$ BE = CE or BE = EC



Now, $BD = \frac{1}{3}BC$

$\Rightarrow BC - DC = \frac{1}{3}BC$

$\Rightarrow BC - \frac{1}{3}BC = DC$

$\Rightarrow DC = \frac{2}{3}BC$

Using Pythagoras theorem in right angled triangles AED and AEC, we get

$AD^2 = AE^2 + DE^2$ ..... (i)

$AC^2 = AE^2 + EC^2$ ..... (ii)

From (i) $AD^2 = AE^2 + DE^2 = AE^2 + (DC - EC)^2$

$= AE^2 + DC^2 + EC^2 - 2DC \times EC$

$\Rightarrow AD^2 = (AE^2 + EC^2) + DC^2 - 2DC \times EC$

$\Rightarrow AD^2 = AC^2 + DC^2 - 2DC \times EC$ [By using (ii)]

$$\Rightarrow AD^2 = AC^2 + \left(\frac{2}{3}BC\right)^2 - 2\left(\frac{2}{3}BC\right) \times \left(\frac{1}{2}BC\right)$$

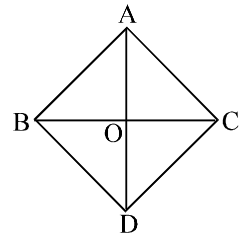
$$\Rightarrow AD^2 = AC^2 + \frac{4}{9}BC^2 - \frac{2}{3}BC^2$$

$$\Rightarrow AD^2 = AB^2 + \frac{4}{9}AB^2 + \frac{2}{3}AB^2$$ ($\because$ Δ ABC is an equilateral triangle, so AB = AC = BC)

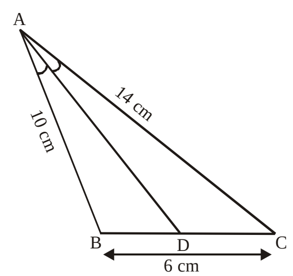
$$\Rightarrow AD^2 = \frac{9AB^2 + 4AB^2 - 6AB^2}{9}$$ $$\Rightarrow AD^2 = \frac{7}{9}AB^2$$

$\Rightarrow 9AD^2 = 7AB^2$

Hence proved.

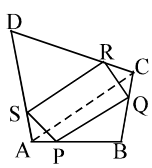


10. In ΔABC, AB = AC and BD $\perp$ AC. Prove that $BD^2 - CD^2 = 2AD.\ CD$
11. In a ΔABC, AD $\perp$ BC and $AD^2 = BD \times CD$. Prove that ΔABC is a right angle.
12. In trapezium ABCD, AB || DC and DC = 2AB. EF drawn parallel to AB cuts AD in F and BC in E such that $\frac{BE}{EC} = \frac{3}{4}$. Diagonal DB intersects EF at G. Prove taht 7FE = 10AB.
13. In the given figure AD is the bisector of $\angle BAC$. If AB = 10 cm, AC = 14 cm and BC 6 cm. Then find BD and DC?

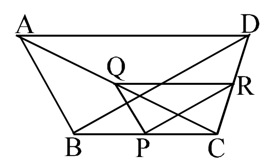


EXERCISE-II

1. Let X be any point on the side BC of a tirangle ABC. If XM, XN are drawn parallel to BA and CA meeting. CA, BA in M, N respectively, MN meets BC produced in T, prove that $TX^2 = (TB)\ (TC)$.
2. Let ABC be a triangle and D and E be two points on side AB such that AD = BE. If DP || BC and EQ || AC, then prove that PQ || AB.
3. ABCD is a quadrilateral. P, Q, R and S are the points of trisection of sides AB, BC, CD, and DA respectively and are adjacent to A and C. Prove that PQRS is a parallelogram.

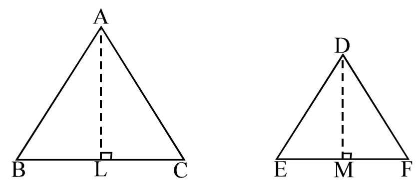


4. ΔABC and ΔDBC line on the same side of the base BC. From a Point P on BC, PQ || AB and PR || BD are drawn. They meet AC in Q and DC in R respectively. Prove that QR || AD.



5. Two poles of height '*a*' metres and '*b*' metres are '*p*' metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{ab}{a+b}$ meters.

6. The areas of two smimilar triangles are 81cm^2 and 49cm^2 respectively. If the altitude of the bigger triangle is 4.5cm, find the corresponding altitude of the smaller triangle.

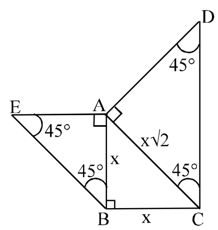


7. Prove that the area of the equilateral described on the side of a square is half the area of the equilateral triangle described on its diagonal.

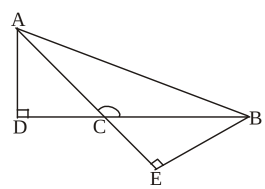
8. In the equilateral triangle ABC, AD is drawn perpendicular to BC, meeting BC in D. Prove that $AD^2 = 3BD^2$.

9. P and Q are the mid points of the sides CA and CB respectively of ΔABC right angled at C. Prove that (i) $4AQ^2 = 4AC^2 + BC^2$ (ii) $4BP^2 = 4BC^2 + AC^2$ (iii) $4(AQ^2 + BP^2) = 5AB^2$.

10. ΔABC is an isosceles triangle right angled at B. Similar triangles ACD and ABE are constructed on sides AC and AB. Find the ratio between the areas of ΔABE and ΔACD.



11. In ΔABC, ∠C is obtuse. AD ⊥ BC produced and BE ⊥ AC produced. Prove that $AB^2 = AC.AE + BC.BD$.

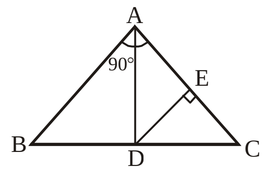


12. In ΔABC, ∠A is obtuse. PB ⊥ AC and QC ⊥ AB. Prove that (i) AB × AQ = AC × AP.

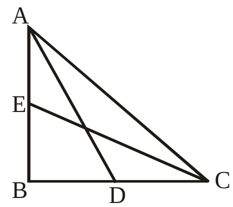
13. ABC is a triangle in which AB = AC and D is any point in BC. Prove that
$AB^2 - AD^2 = BD \cdot CD$.

14. In A be the area of a right triangle and b one of the sides containing the right angle, prove that the length of the altitude on the hypotenuse is $\frac{2Ab}{\sqrt{b^4 + 4A^2}}$.

15. In figure, ∠BAC = 90°, AD is its bisector. If DE ⊥ AC, prove that DE × (AB + AC) = AB × AC.



16. In figure, ABC is a right triangle right-angled at B. AD and CE are the two medians drawn from A and C respectively. If AC = 5 cm and AD = $\frac{3\sqrt{5}}{2}$ cm, find the length of CE.

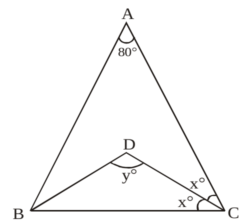


EXERCISE-III

SECTION-A

- **Multiple choice question with one correct answers**

1. In the given figure $\angle A = 80°$, B = 60°, C = 2x° and $\angle BDC = y°$, BD and CD bisect angles B and C respectively. The values of x and y, respectively, are



(A) 15°, 70° (B) 10°, 160° (C) 20°, 130° (D) 20°, 125°

2. If a + b + c = 2s, then the value of $(s-a)^2 + (s-b)^2 + (s-c)^2$ will be:

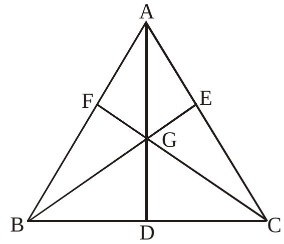
(A) $s^2 + a^2 + b^2 + c^2$ (B) $a^2 + b^2 + c^2 - s^2$

(C) $s^2 - a^2 - b^2 - c^2$ (D) $4s^2 - a^2 - b^2 - c^2$

3. If D is a point on the side BC = 12 cm of a ΔABC such that BD = 9 cm and $\angle ADC = \angle BAC$, then the length of AC is equal to:

(A) 9 cm (B) 6cm (C) $6\sqrt{3}$ (D) 3 cm

4. In ΔABC medians BE and CF intersect at G. If the straightline AGD meets BC at D in such a way that GD = 1.5 cm, then the length of AD is :



(A) 2.5 cm (B) 3.0 cm (C) 4.00 cm (D) 4.5 cm

5. The side of an equilateral triangle is $20\sqrt{3}$ cm. The numerical value of the radius of the circle circumscribing the triangle is :

(A) 20 cm (B) $20\sqrt{3}$ cm (C) 20π cm (D) $\frac{20}{\pi}$

6. If ΔABC is a right angled triangle with $\angle A = 90°$, AN is perpendicular to BC, BC = 12 cm and AC = 6 cm, then the ratio of $\frac{\text{area } \Delta ANC}{\text{area } \Delta ABC}$:

(A) 1 : 3 (B) 1 : 2 (C) 1 : 4 (D) 1 : 8

7. The area of the largest triangle inscribed in a semi - circle of radius R is :

(A) $2R^2$ (B) R^2 (C) $\frac{1}{2}R^2$ (D) $\frac{3}{2}R^2$

8. In a triangle ABC, then sum of the exterior angles at B and C is equal to :

(A) $180° - \angle BAC$ (B) $180° + \angle BAC$ (C) $180° - 2\angle BAC$ (D) $180° + 2\angle BAC$

9. In ΔABC, $\angle B = 3x$, $\angle A = x$, $\angle C = y$ and $3y - 5x = 30$, then the triangle is ;

(A) isosceles (B) equlateral (C) right angled (D) scalene

10. The internal bisectors of $\angle B$ and $\angle C$ of ΔABC meet at O. If $\angle A = 80°$, then $\angle BOC$ is:

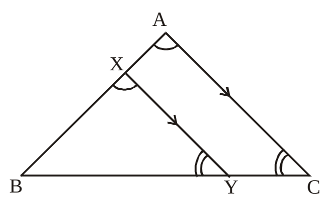
(A) 50° (B) 100° (C) 130° (D) 160°

SECTION-B

- **Match the following (one to one)**

Column-I and **column-II** contains **four** entries each. Entries of column-I are to be matched with some entries of column-II. Only One entries of column-I may have the matching with the some entries of column-II and one entry of column-II Only one matching with entries of column-I

In the figure, the line segment xy is parallel to the side AC of Δ ABC and it divides the triangle into two parts of equal areas, then match the column



1. Column I	Column II
(A) AB : XB	(P) $\sqrt{2}$: 1
(B) ar (Δ ABC) : ar (Δ XBY)	(Q) 2 : 1
(C) AX : AB	(R) $\left(\sqrt{2}-1\right)^2 : \sqrt{2}$
(D) ∠X : ∠A	(S) 1 : 1

EXERCISE-IV

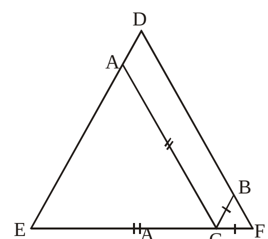
SECTION-A

- **Multiple choice question with one correct answers**

1. The areas of two similar triangles are 12 cm^2 and 48 cm^2. If the height of the samller one is 2.1 cm, then the corresponding height of the bigger triangle is:

   (A) 12.6 cm (B) 8.4 cm (C) 4.2 cm (D) 1.05 cm

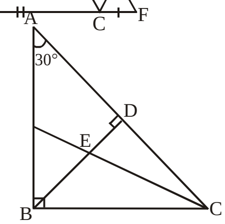
2. In a triangle DEF shown in given figure, points A, B and C are taken on DE, DF and EF respectively, such that EC = AC and CF = BC. If angle D = 40°, then what is angle ACB in degrees?



   (A) 140 (B) 70

   (C) 100 (D) None of these

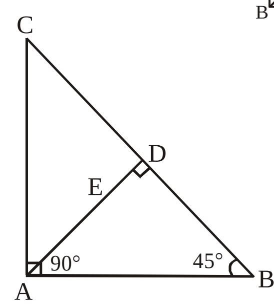
3. AB ⊥ BC, BD ⊥ AC and CE bisects ∠C. If A = 30°. Then, what is ∠CED?



   (A) 30° (B) 60°

   (C) 45° (D) 65°

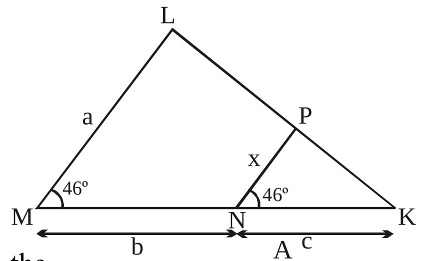
\4. In ΔABC, ∠A = 90°, AC ⊥ BC and ∠B = 45°. If AB = x, then the value of AD in terms of x is :



   (A) $\frac{x}{2}$ (B) $\frac{\sqrt{x}}{2}$

   (C) $\frac{x}{\sqrt{2}}$ (D) $\sqrt{\frac{x}{2}}$

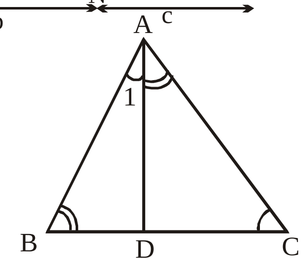
5. Express x in terms of a, b, and c.



(A) $x = \frac{ac}{b+c}$ (B) $x = \frac{bc}{a+c}$

(C) $x = \frac{b+c}{ac}$ (D) $x = \frac{ab}{a+c}$

6. In Δ ABC, if AD ⊥ BC and $AD^2 = BD \times DC$. Then find the angle ∠BAC = ?



(A) 60º (B) 90º

(C) 30º (D) None of this

7. PB and QA are perpendiculars to segment AB. If PO = 5 cm, QO = 7 cm and area Δ POB = 150 cm^2, find the area of Δ QOA.

(A) 294 cm^2 (B) 150 cm^2 (C) 250 cm^2 (D) 210 cm^2

8. The corresponding altitude of two similar triangles are 6 cm and 9 cm respectively. Find the ratio of their areas.

(A) 9 : 4 (B) 3 : 2 (C) 4 : 9 (D) 8 : 16

SECTION-B

- **Multiple choice question with one or more than one correct answers**

1. ABC is a triangle right-angled at C with BC = a and AC = b. If p is the length of the prependicular from C on AB then.

(A) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ (B) $p^2 = \frac{a^2b^2}{a^2+b^2}$ (C) $\frac{2}{p^2} = \frac{a^2b^2}{a^2+b^2}$ (D) $p^2 = \frac{a^2b^2}{a^2-b^2}$

2. ABC is a right triangle, right angled at C, let BC = a, CA = b, AB = c and let p be the length of perpendicular from C on AB. Then which of the following is correct?

(A) CP = ab (B) $\frac{a}{c} = \frac{p}{b}$ (C) cb = ap (D) None of these

3. Through the mid-point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting AC in AD produced ME. Then which of the following is correct?

(A) BL = 2EL (B) EL = 2BL (C) BE = ½EL (D) BL = EL

SECTION-C

- **Comprehension**

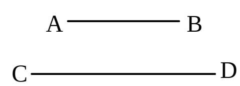
Δ ACB ~ Δ APQ. If BC = 10 cm, PQ = 5 cm, BA = 6.5 cm and AP = 2.8 cm.

4 SIMILAR TRIANGLES

4.1 INTRODUCTION

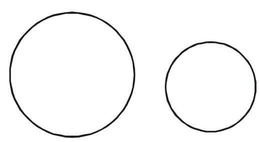
In previous classes, we have learnt about the congruency of two geometric figure. In this chapter we shall learn about these geometric figures. Which have the same but not necessary have the same size. These kind of geometric figures are known as simmilar figures. So the congruent figures are always similar figures but similar figures need not be congruence figures.

(i) Two line segment are similar



(The two line segment are congruent if they have the same length)

(ii) Two circles are similar _____________. (The two circles are congruent. If they have the same radius)



4.2 SIMILAR TRIANGLE

4.2.1 Similar Polygons

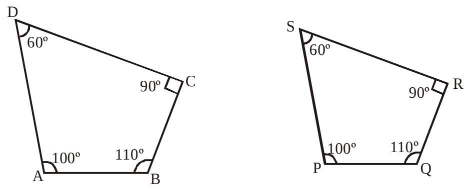
Two polygons of the same number of sides are said to be similar. If

(i) Their corresponding angles are equal

(ii) Their corresponding sides are in the same ratio

Illustration 1

If two polygons ABCD and PQRS are similar then



Solution

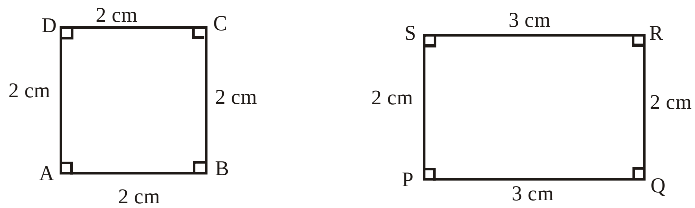
By the definition

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CD}{RS} = \frac{DA}{SP} = 2$$

So corresponding sides are proportional.

Therefor quadrilateral ABCD and PQRS are similar

Illustration 2



Solution

Clarly A square ABCD and rectengle PQRS are equiangular.

But corresponding sides of square ABCD and rectangle PQRS are not proportional.

Therefor square ABCD and rectangle PQRS are not similar.

Remark: If one polygon is similar to a second polygon and the second polygon is similar to a third polygon, then the first polygon is similar to the third polygon.

4.3 SIMILARITY OF TRIANGLES

Two triangles are said to be similar if

(i) Their corresponding angles are equal (or triangles are equiangular)

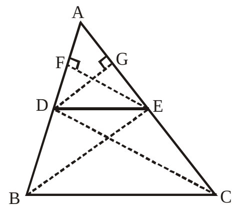
(ii) Their corresponding sides are in the same ratio (or proportional)

Remark: equiangular triangles means that the corresponding angles of the triangles are equal.

Theorem 1: (Basic proportionality theorem or thales theorem)

If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.

$$\frac{AD}{DB} = \frac{AE}{EC}$$



4.3.1 Carollary

If in a ΔABC, a line DE || BC, intersects AB in D and AC in E then

(i) $\frac{AB}{AD} = \frac{AC}{AE}$ (ii) $\frac{AD}{DB} = \frac{AC}{EC}$

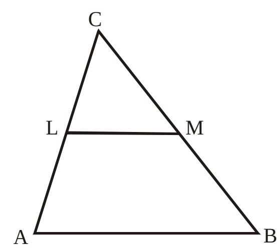
(ii) from the basic proportionality theorem

Theorem–2: Converse of basic proportionality Theorem

If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

Illustration 3

LM || AB. If AL = x – 3, AC = 2x, BM = x – 2, BC = 2x + 3 find the value of x?



Solution

In Δ ABC we have

LM || AB

$$\frac{AL}{LC} = \frac{BM}{MC} \quad \Rightarrow \quad \frac{AL}{AC-AL} = \frac{BM}{BC-BM}$$

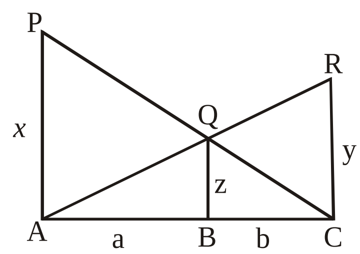
$$\frac{x-3}{2x-(x-3)} = \frac{x-2}{(2x+3)-(x-2)}$$

$$\Rightarrow \frac{x-3}{x+3} = \frac{x-2}{x+5} \quad \Rightarrow (x-3)(x+5) = (x-2)(x+3)$$

$$\Rightarrow x^2 + 2x - 15 = x^2 + x - 6 \quad \Rightarrow x = 9$$

Illustration 4

In the given figure PA, QB and RC each is perpendicular to AC such that PA = x, RC = y, QB = z, AB = a and BC = b. Prove that $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$.



Solution

PA ⊥ AC and QB ⊥ AC ⇒ QB || PA

Thus in Δ PAC, QB || PA

so Δ QBC ~ Δ PAC

$\therefore \quad \frac{QB}{PA}=\frac{BC}{AC} \quad \Rightarrow \frac{z}{x}=\frac{b}{a+b}$ ..... (i) (by the property of similar triangle)

In Δ RAC, QB || RC, so Δ QBC ~ Δ RAC

$\therefore \quad \frac{QB}{RC}=\frac{AB}{AC} \quad \Rightarrow \frac{z}{y}=\frac{b}{a+b}$ ..... (ii) (by the property of similar triangle)

Now from (i) and (ii) we get

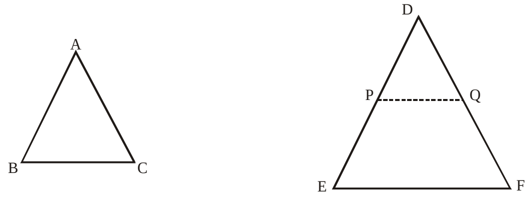
$$\frac{z}{x}+\frac{z}{y}=\left(\frac{b}{a+b}+\frac{a}{a+b}\right)=1$$

$$\frac{z}{x}+\frac{z}{y}=1$$

$$\left(\frac{1}{x}+\frac{1}{y}=\frac{1}{z}\right)$$

THEOREM–3: (AAA - SIMILARITY)

If in two triangle. The corresponding angles are equal, then their corresponding sides are proportional and hence the triangles are similar.



4.4.1 Corollary (AA Similarity)

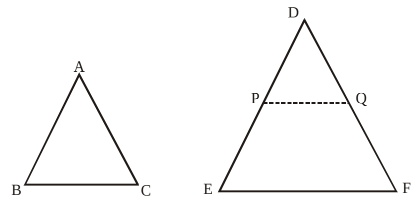
If two angles of one triangle are respectively equal to two angles of another triangle than the two triangles are similar.

Theorem–4: (SSS-Similarity)

If the corresponding sides of two triangles are proportional then their corresponding angles are equal and hence the two triangles are similar.

Given :

Δ ABC and Δ DEF & $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$



Theorem–5: (SAS-similarity)

If one angles of a triangles is equal to one angle of the other triangle and the sides including these angles are proportional then two triangles are similar.

ΔABC and ΔDEF

$\angle A = \angle D$ & $\dfrac{AB}{DE} = \dfrac{AC}{DF}$

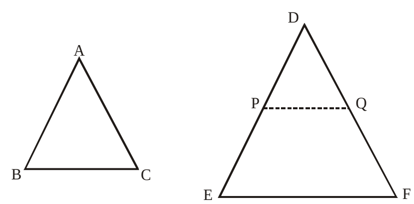
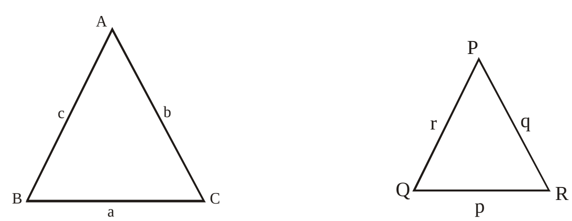


Illustration 5

Prove that the ratio of the perimeters of two similar triangles is the same as the ratio of their corresponding sides.

Solution



Given: Δ ABC and Δ PQR

BC = a, CA = b, AB = c

and QR = p, RP = q, PQ = r

Also Δ ABC ~ Δ PQR

To prove: $\dfrac{a}{p} = \dfrac{b}{q} = \dfrac{c}{r} = \dfrac{a+b+c}{p+q+r}$

Proof: Since Δ ABC and Δ PQR are similar, there for their corresponding sides are proportional

$$\therefore \qquad \frac{a}{p} = \frac{b}{q} = \frac{c}{r} = k \text{ (say)}$$

a = kp, b = kq, c = kr

$$\therefore \quad \frac{\text{perimeter of } \Delta ABC}{\text{perimeter of } \Delta PQR} = \frac{a+b+c}{p+q+r} = \frac{kp+kq+kr}{p+q+r}$$

$$= \frac{k(p+q+r)}{p+q+r} = k \qquad \text{..... (ii)}$$

from (i) and (ii) we get

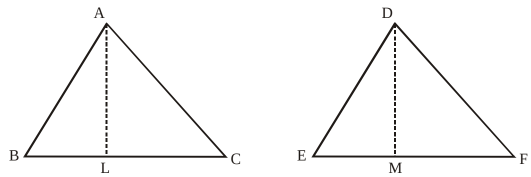
$$\frac{a}{p} = \frac{b}{q} = \frac{c}{r} = \frac{a+b+c}{p+q+r} = \frac{\text{perimeter of } \Delta ABC}{\text{perimeter of } \Delta PQR}$$

4.4 AREAS OF SIMILAR TRIANGLES

Theorem–6: The ratio of the area of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Given: Δ ABC ~ Δ DEF

To Prove : $$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2}$$



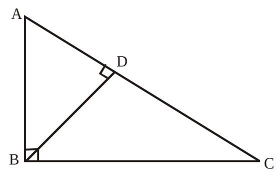
4.5 PYTHAGORAS THEOREM

Theorem–7: In a right triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Given : In Δ ABC

$\angle ABC = 90^o$

To prove: $AC^2 = AB^2 + BC^2$



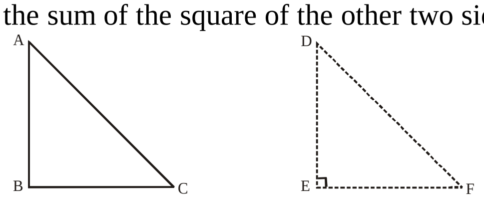
Theorem–8: Converse of Pythagoras theorem

In a triangle , if the square of one side is equal to the sum of the square of the other two sides then the angle opposite to the first side is right angle.

In Δ ABC

$AC^2 = AB^2 + BC^2$

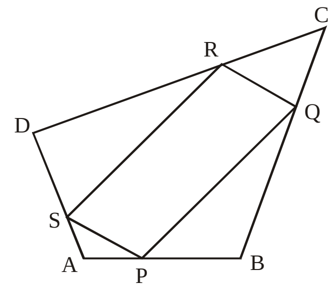
$\angle B = 90^o$



Solved Examples

Example 1

In the adjoining figure. ABCD is a quadrilateral and P, Q, R, S are the points of trisection of the sides AB, BC, CD and DA respectively. Prove that PQRS is a parallelogram.



Solution

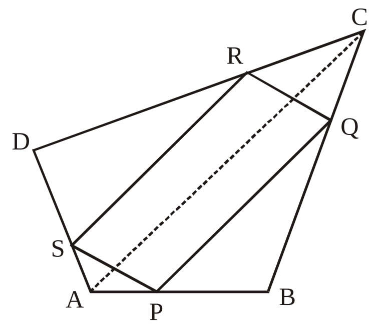
Here, ABCD is a quadrilateral. Since R and S are points of trisection of sides CD and DA respectively.

$\therefore$ CD = 3CR or CR + DR = 3CR

or DR = 2CR or $\frac{DR}{RC} = \frac{2}{1}$ and

AD = 3AS or AS + SD = 2AS

or DS = 2AS or $\frac{DS}{SA} = \frac{2}{1}$



$\Rightarrow \frac{DS}{SA} = \frac{DR}{RC}$

$\therefore$ By converse of basic proportionality theorem,

In Δ DAC, $\frac{DS}{SA} = \frac{DR}{RC}$ $\Rightarrow$ SR || AC

Similarly, PQ || AC

$\therefore$ SR || AC and PQ || AC $\Rightarrow$ SR || PQ

$\Rightarrow$ Similarly one can prove that PS || QR

Hence, PQRS is a parallelogram.

Example 2

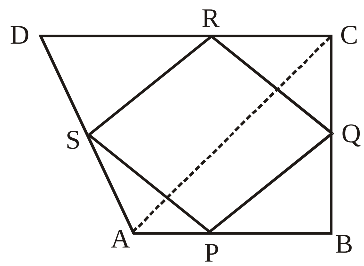
Prove that the line segments joining the mid-points of the adjacent sides of a quadrilateral form a parallelogram.

Solution

Let P, Q, R and S respectively be the mid-points of the sides AB, BC, CD and DA of the quadrilateral ABCD.

Join PQ, QR, RS and SP.

Also, join AC,



Since S and R are the mid-points of DA and DC respectively

$\therefore$ DS = SA and DR = RC

$\Rightarrow \frac{DS}{SA} = 1$ and $\frac{DR}{RC} = 1$

$\Rightarrow \frac{DS}{SA} = \frac{DR}{RC}$

$\therefore$ In Δ DAC, $\frac{DS}{SA} = \frac{DR}{RC}$ $\Rightarrow$ SR $\parallel$ AC ..... (i) (By converse of basic proportionality theorem)

Since Q and P are the mid-points of BC and BA respectively.

$\therefore$ BQ = QC and BP = PA

$\Rightarrow \frac{BQ}{QC} = 1$ and $\frac{BP}{PA} = 1$

$\Rightarrow \frac{BQ}{QC} = \frac{BP}{PA}$

$\therefore$ In Δ BCA, $\frac{BQ}{QC} = \frac{BP}{PA}$

$\Rightarrow$ QP $\parallel$ CA or PQ $\parallel$ AC ..... (ii) (By converse of basic proportionality theorem)

From (i) and (ii), we have

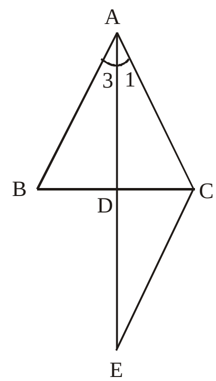
PQ $\parallel$ SR

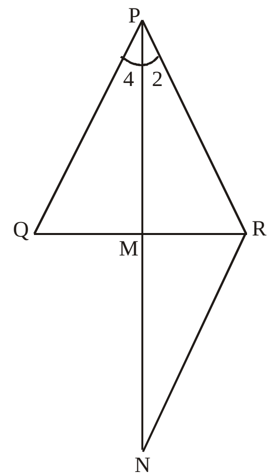
Similarly, PS $\parallel$ QR

Hence, PQRS is a parallelogram.

Example 3

Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that Δ ABC ~ ΔPQR.





Solution

Since sides AB and AC and median AD of a triangle ABC are respectively proportinal to sides PQ and PR and median PM of another triangle PQR.

$\therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

Now, produce AD to E and PM to N such that AD = DE and PM = MN.

Join EC and NR

In triangles ADB and EDC,

AD = DE (construction)

BD = CD ($\because$ AD is median)

$\angle$ADB = $\angle$EDC (Vertically opposite angles)

$\therefore$ By S.A.S. congruency criteria,

Δ ADB $\cong$ Δ EDC

$\Rightarrow$ AB = EC ($\because$ corresponding parts of congruent triangles are equal)

Now, in triangles PMQ and NMR,

PM = MN (construction)

QM = RM ($\because$ PM is median)

$\angle$PMQ = $\angle$NMR (vertically opposite angles)

∴ By S.A.S. congruency criteria,

$\Delta PMQ \cong \Delta NMR$

$\Rightarrow$ PQ = NR (∵ corresponding parts of congruent triangles are equal)

Now, $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

$\Rightarrow \frac{EC}{NR} = \frac{AC}{PR} = \frac{AD}{PM}$ [∵ AB = EC and PQ = NR (proved above)]

$\Rightarrow \frac{EC}{NR} = \frac{AC}{PR} = \frac{2AD}{2PM} \Rightarrow \frac{EC}{NR} = \frac{AC}{PR} = \frac{AE}{PN}$

$\Rightarrow \Delta AEC \sim \Delta PNR$ (By using S.S.S. similarity criteria)

$\Rightarrow \angle 1 = \angle 2$ (∵ Similar triangles are equiangular)

Similarly, we can prove that $\angle 3 = \angle 4$

∴ $\angle 1 + \angle 3 = \angle 2 + \angle 4$

$\Rightarrow \angle BAC = \angle QPR$

Now, in triangles ABC and PQR,

$\angle BAC = \angle QPR$ (prove above)

$\frac{AB}{PQ} = \frac{AC}{PR}$ (given)

∴ By S.A.S. similarity criteria,

$\Delta ABC \sim \Delta PQR$

Hence proved.

Example 4

Two poles of heights a metres and b metres are p metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{ab}{a+b}$ ***metres.***

Solution

Let AL and BM represent the two poles of heights a metres and b metres repectively.

Since the poles are p metres apart.

∴ LM = p metres

Let O be the point of intersection of the lines AM and BL

From O, draw On perpendicular on LM

Let ON = h metres and LN = x metres

∴ Nm = LM – LN = (p – x) metres

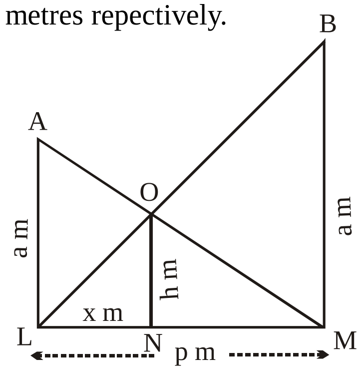
In triangles LMB and LNO,

$\angle LMB = \angle LNO$ (Each = 90°)

$\angle L = \angle L$ (common)

∴ By A.A. similarity criteria

$\Delta LMB \sim \Delta LNO$



$\Rightarrow \quad \frac{LM}{LN} = \frac{MB}{NO} \qquad \Rightarrow \frac{p}{x} = \frac{b}{h} \qquad \Rightarrow x = \frac{ph}{b}$ ..... (i)

In triangles MLA and MNO,

$\angle MLA = \angle MNO$ (each 90º)

$\angle LMA = \angle NMO$ (common)

$\therefore$ By A.A. similarity criteria,

Δ MLA ~ Δ MNO

$\Rightarrow \quad \frac{ML}{MN} = \frac{LA}{NO} \qquad \Rightarrow \frac{p}{p-x} = \frac{a}{h} \qquad \Rightarrow p - x = \frac{ph}{a} \quad \Rightarrow x = p - \frac{ph}{a}$ ..... (ii)

From (i) and (ii), we have

$$\frac{ph}{b} = p - \frac{ph}{a}$$

$\Rightarrow \quad \frac{ph}{a} + \frac{ph}{b} = p \qquad \Rightarrow ph\left(\frac{1}{a} + \frac{1}{b}\right) = p \qquad \Rightarrow h\left(\frac{b+a}{ab}\right) = 1$

$\Rightarrow \quad h = \frac{ab}{a+b}$

Hence, the required height is $\frac{ab}{a+b}$ metres.

Example 5

In an equilateral triangle ABC, D is a point on side BC such that BD = 1/3 BC. Prove that $9AD^2 = 7AB^2$.

Solution

In equilateral Δ ABC, D is a point on side BC such that BD = $\frac{1}{3}$BC.

From A, draw AE $\perp$ BC.

Also, join AD

Now, in right angled triangles AEB and AEC,

$\angle B = \angle C$ (each 60º)

$\angle AEB = \angle AEC$ (each 90º)

$\therefore$ By A.A. similarity criteria,

Δ AEB ~ Δ AEC

$\Rightarrow \quad \frac{AB}{AC} = \frac{AE}{AE} = \frac{BE}{CE}$ ($\because$ corresponding sides of similar triangles are proportional)

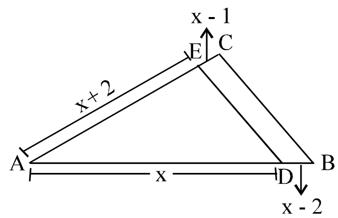
$\Rightarrow \quad \frac{AB}{AC} = 1 = \frac{BE}{CE}$

$\Rightarrow \quad \frac{BE}{CE} = 1$

*****

EXERCISE-I

1. In the given figure, DE || BC. If AD = x, DB = x – 2, AE = x + 2 and EC = x – 1, find the value of x.



2. ABCD is a parallelogram, P is a point on side BC and DP, when produced meets AB produced at L. Prove that (i) $\frac{DP}{PL} = \frac{DC}{BL}$, (ii) $\frac{DL}{DP} = \frac{AL}{DC}$.

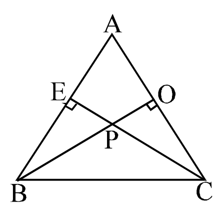
3. In a ΔABC , AD is the bisector of $\angle$A, meeting side BC at D. If AC = 4.2cm, DC = 6cm, BC = 10cm, find AB.

4. In ΔABC, if AD is the bisector of $\angle$A, Prove that $\frac{Area(\Delta ABD)}{Area(\Delta ACD)} = \frac{AB}{AC}$.

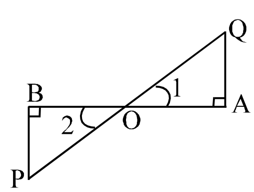
5. In ΔABC, D is the mid point of BC and ED is the bisector of $\angle$ADB and EF is drawn parallel to BC cutting AC in F. Prove that $\angle$EDF is a right angle.

6. Two triangles ΔBAC and ΔBDC, right angled at A and D respectively, are drawn on the same base BC and on the same side of BC. If AC and DB intersect at P, Prove that AP × PC = DP × PB.

7. In the given figure, considering the tiangles BEP and CPO, prove that BP × PD = EP × PC.



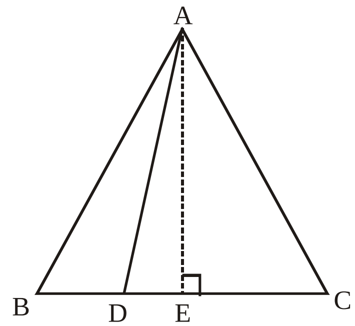
8. In the given figure PB and QA perpendicular to segment AB. If PO = 5cm, QO = 7cm and area ΔPOB = 150cm^2, find the area of ΔQOA.



9. In the given figure, ΔABC and ΔDBC are two triangles on the same base BC. If AD intersects BC at O, prove

that $\frac{area(\Delta ABC)}{area(\Delta DBC)} = \frac{AO}{DO}$.

$\Rightarrow$ BE = CE or BE = EC



Now, $BD = \frac{1}{3}BC$

$\Rightarrow BC - DC = \frac{1}{3}BC$

$\Rightarrow BC - \frac{1}{3}BC = DC$

$\Rightarrow DC = \frac{2}{3}BC$

Using Pythagoras theorem in right angled triangles AED and AEC, we get

$AD^2 = AE^2 + DE^2$ ..... (i)

$AC^2 = AE^2 + EC^2$ ..... (ii)

From (i) $AD^2 = AE^2 + DE^2 = AE^2 + (DC - EC)^2$

$= AE^2 + DC^2 + EC^2 - 2DC \times EC$

$\Rightarrow AD^2 = (AE^2 + EC^2) + DC^2 - 2DC \times EC$

$\Rightarrow AD^2 = AC^2 + DC^2 - 2DC \times EC$ [By using (ii)]

$\Rightarrow AD^2 = AC^2 + \left(\frac{2}{3}BC\right)^2 - 2\left(\frac{2}{3}BC\right) \times \left(\frac{1}{2}BC\right)$

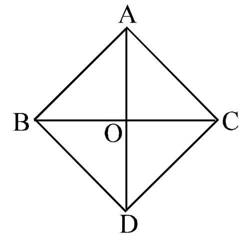
$\Rightarrow AD^2 = AC^2 + \frac{4}{9}BC^2 - \frac{2}{3}BC^2$

$\Rightarrow AD^2 = AB^2 + \frac{4}{9}AB^2 + \frac{2}{3}AB^2$ ($\because$ Δ ABC is an equilateral triangle, so AB = AC = BC)

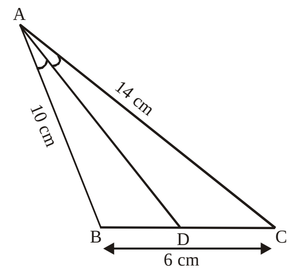
$\Rightarrow AD^2 = \frac{9AB^2 + 4AB^2 - 6AB^2}{9}$ $\Rightarrow AD^2 = \frac{7}{9}AB^2$

$\Rightarrow 9AD^2 = 7AB^2$

Hence proved.

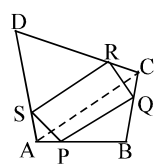


10. In ΔABC, AB = AC and BD $\perp$ AC. Prove that $BD^2 - CD^2 = 2AD.CD$
11. In a ΔABC, AD $\perp$ BC and $AD^2 = BD \times CD$. Prove that ΔABC is a right angle.
12. In trapezium ABCD, AB || DC and DC = 2AB. EF drawn parallel to AB cuts AD in F and BC in E such that $\frac{BE}{EC} = \frac{3}{4}$. Diagonal DB intersects EF at G. Prove taht 7FE = 10AB.
13. In the given figure AD is the bisector of $\angle$BAC. If AB = 10 cm, AC = 14 cm and BC 6 cm. Then find BD and DC?

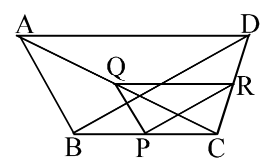


EXERCISE-II

1. Let X be any point on the side BC of a tirangle ABC. If XM, XN are drawn parallel to BA and CA meeting. CA, BA in M, N respectively, MN meets BC produced in T, prove that TX^2 = (TB) (TC).
2. Let ABC be a triangle and D and E be two points on side AB such that AD = BE. If DP || BC and EQ || AC, then prove that PQ || AB.
3. ABCD is a quadrilateral. P, Q, R and S are the points of trisection of sides AB, BC, CD, and DA respectively and are adjacent to A and C. Prove that PQRS is a parallelogram.

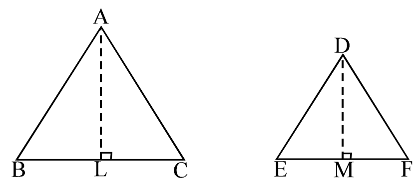


4. ΔABC and ΔDBC line on the same side of the base BC. From a Point P on BC, PQ || AB and PR || BD are drawn. They meet AC in Q and DC in R respectively. Prove that QR || AD.



5. Two poles of height '*a*' metres and '*b*' metres are '*p*' metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{ab}{a+b}$ meters.

6. The areas of two smimilar triangles are $81cm^2$ and $49cm^2$ respectively. If the altitude of the bigger triangle is 4.5cm, find the corresponding altitude of the smaller triangle.

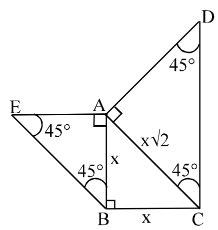


7. Prove that the area of the equilateral described on the side of a square is half the area of the equilateral triangle described on its diagonal.

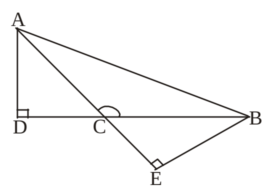
8. In the equilateral triangle ABC, AD is drawn perpendicular to BC, meeting BC in D. Prove that $AD^2 = 3BD^2$.

9. P and Q are the mid points of the sides CA and CB respectively of ΔABC right angled at C. Prove that (i) $4AQ^2 = 4AC^2 + BC^2$ (ii) $4BP^2 = 4BC^2 + AC^2$ (iii) $4(AQ^2 + BP^2) = 5AB^2$.

10. ΔABC is an isosceles triangle right angled at B. Similar triangles ACD and ABE are constructed on sides AC and AB. Find the ratio between the areas of ΔABE and ΔACD.



11. In ΔABC, ∠C is obtuse. AD ⊥ BC produced and BE ⊥ AC produced. Prove that $AB^2 = AC.AE + BC.BD$.

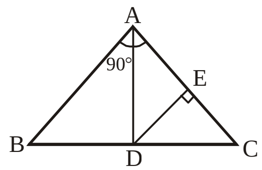


12. In ΔABC, ∠A is obtuse. PB ⊥ AC and QC ⊥ AB. Prove that (i) AB × AQ = AC × AP.

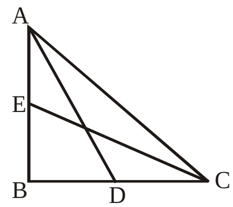
13. ABC is a triangle in which AB = AC and D is any point in BC. Prove that
$AB^2 - AD^2 = BD \cdot CD$.

14. In A be the area of a right triangle and b one of the sides containing the right angle, prove that the length of the altitude on the hypotenuse is $\frac{2Ab}{\sqrt{b^4 + 4A^2}}$.

15. In figure, ∠BAC = 90°, AD is its bisector. If DE ⊥ AC, prove that DE × (AB + AC) = AB × AC.



16. In figure, ABC is a right triangle right-angled at B. AD and CE are the two medians drawn from A and C respectively. If AC = 5 cm and AD = $\frac{3\sqrt{5}}{2}$ cm, find the length of CE.

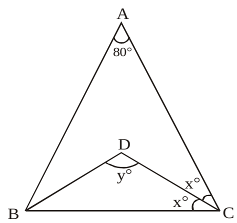


EXERCISE-III

SECTION-A

- **Multiple choice question with one correct answers**

1. In the given figure ∠A = 80°, B = 60°, C = 2x° and ∠BDC = y°, BD and CD bisect angles B and C respectively. The values of x and y, respectively, are



(A) 15°, 70° (B) 10°, 160° (C) 20°, 130° (D) 20°, 125°

2. If a + b + c = 2s, then the value of $(s - a)^2 + (s - b)^2 + (s - c)^2$ will be:

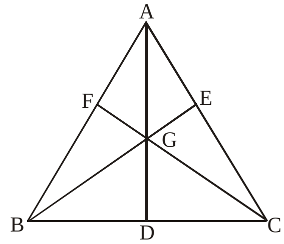
(A) $s^2 + a^2 + b^2 + c^2$ (B) $a^2 + b^2 + c^2 - s^2$

(C) $s^2 - a^2 - b^2 - c^2$ (D) $4s^2 - a^2 - b^2 - c^2$

3. If D is a point on the side BC = 12 cm of a ΔABC such that BD = 9 cm and ∠ADC = ∠BAC, then the length of AC is equal to:

(A) 9 cm (B) 6cm (C) $6\sqrt{3}$ (D) 3 cm

4. In ΔABC medians BE and CF intersect at G. If the straightline AGD meets BC at D in such a way that GD = 1.5 cm, then the length of AD is :



(A) 2.5 cm (B) 3.0 cm (C) 4.00 cm (D) 4.5 cm

5. The side of an equilateral triangle is $20\sqrt{3}$ cm. The numerical value of the radius of the circle circumscribing the triangle is :

(A) 20 cm (B) $20\sqrt{3}$ cm (C) 20 π cm (D) $\frac{20}{\pi}$

6. If ΔABC is a right angled triangle with $\angle A = 90°$, AN is perpendicular to BC, BC = 12 cm and AC = 6 cm, then the ratio of $\frac{\text{area } \Delta ANC}{\text{area } \Delta ABC}$:

(A) 1 : 3 (B) 1 : 2 (C) 1 : 4 (D) 1 : 8

7. The area of the largest triangle inscribed in a semi - circle of radius R is :

(A) $2R^2$ (B) R^2 (C) $\frac{1}{2}R^2$ (D) $\frac{3}{2}R^2$

8. In a triangle ABC, then sum of the exterior angles at B and C is equal to :

(A) $180° - \angle BAC$ (B) $180° + \angle BAC$ (C) $180° - 2\angle BAC$ (D) $180° + 2\angle BAC$

9. In ΔABC, $\angle B = 3x$, $\angle A = x$, $\angle C = y$ and $3y - 5x = 30$, then the triangle is ;

(A) isosceles (B) equlateral (C) right angled (D) scalene

10. The internal bisectors of $\angle B$ and $\angle C$ of ΔABC meet at O. If $\angle A = 80°$, then $\angle BOC$ is:

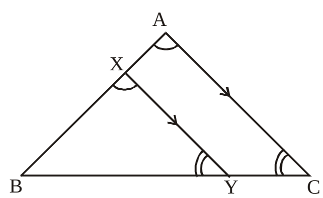
(A) 50° (B) 100° (C) 130° (D) 160°

SECTION-B

- **Match the following (one to one)**

Column-I and **column-II** contains **four** entries each. Entries of column-I are to be matched with some entries of column-II. Only One entries of column-I may have the matching with the some entries of column-II and one entry of column-II Only one matching with entries of column-I

In the figure, the line segment xy is parallel to the side AC of Δ ABC and it divides the triangle into two parts of equal areas, then match the column



1. Column I	Column II
(A) AB : XB	(P) $\sqrt{2}$: 1
(B) ar (Δ ABC) : ar (Δ XBY)	(Q) 2 : 1
(C) AX : AB	(R) $\left(\sqrt{2}-1\right)^2 : \sqrt{2}$
(D) ∠X : ∠A	(S) 1 : 1

EXERCISE-IV

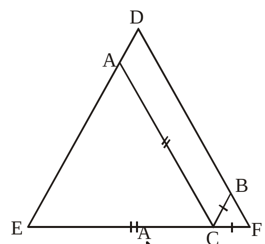
SECTION-A

- **Multiple choice question with one correct answers**

1. The areas of two similar triangles are 12 cm^2 and 48 cm^2. If the height of the samller one is 2.1 cm, then the corresponding height of the bigger triangle is:

(A) 12.6 cm (B) 8.4 cm (C) 4.2 cm (D) 1.05 cm

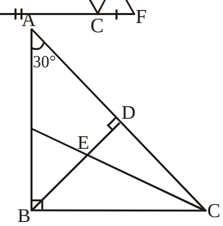
2. In a triangle DEF shown in given figure, points A, B and C are taken on DE, DF and EF respectively, such that EC = AC and CF = BC. If angle D = 40°, then what is angle ACB in degrees?



(A) 140 (B) 70

(C) 100 (D) None of these

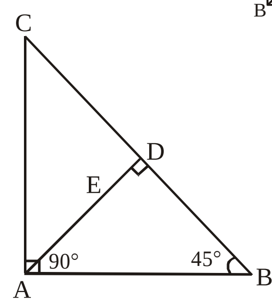
3. AB ⊥ BC, BD ⊥ AC and CE bisects ∠C. If A = 30°. Then, what is ∠CED?



(A) 30° (B) 60°

(C) 45° (D) 65°

\4. In ΔABC, ∠A = 90°, AC ⊥ BC and ∠B = 45°. If AB = x, then the value of AD in terms of x is :



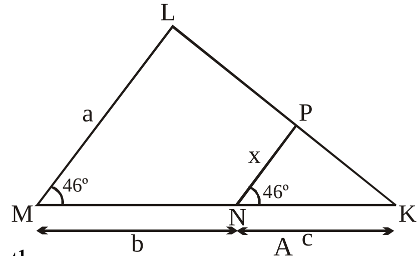
(A) $\frac{x}{2}$ (B) $\frac{\sqrt{x}}{2}$

(C) $\frac{x}{\sqrt{2}}$ (D) $\sqrt{\frac{x}{2}}$

5. Express x in terms of a, b, and c.

(A) $x = \frac{ac}{b + c}$ (B) $x = \frac{bc}{a + c}$

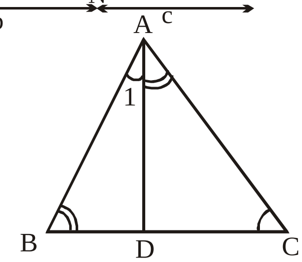
(C) $x = \frac{b + c}{ac}$ (D) $x = \frac{ab}{a + c}$



6. In Δ ABC, if AD ⊥ BC and $AD^2 = BD \times DC$. Then find the angle ∠BAC = ?

(A) 60º (B) 90º

(C) 30º (D) None of this



7. PB and QA are perpendiculars to segment AB. If PO = 5 cm, QO = 7 cm and area Δ POB = 150 cm^2, find the area of Δ QOA.

(A) 294 cm^2 (B) 150 cm^2 (C) 250 cm^2 (D) 210 cm^2

8. The corresponding altitude of two similar triangles are 6 cm and 9 cm respectively. Find the ratio of their areas.

(A) 9 : 4 (B) 3 : 2 (C) 4 : 9 (D) 8 : 16

SECTION-B

- **Multiple choice question with one or more than one correct answers**

1. ABC is a triangle right-angled at C with BC = a and AC = b. If p is the length of the prependicular from C on AB then.

(A) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ (B) $p^2 = \frac{a^2b^2}{a^2 + b^2}$ (C) $\frac{2}{p^2} = \frac{a^2b^2}{a^2 + b^2}$ (D) $p^2 = \frac{a^2b^2}{a^2 - b^2}$

2. ABC is a right triangle, right angled at C, let BC = a, CA = b, AB = c and let p be the length of perpendicular from C on AB. Then which of the following is correct?

(A) CP = ab (B) $\frac{a}{c} = \frac{p}{b}$ (C) cb = ap (D) None of these

3. Through the mid-point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting AC in AD produced ME. Then which of the following is correct?

(A) BL = 2EL (B) EL = 2BL (C) BE = ½EL (D) BL = EL

SECTION-C

- **Comprehension**

Δ ACB ~ Δ APQ. If BC = 10 cm, PQ = 5 cm, BA = 6.5 cm and AP = 2.8 cm.