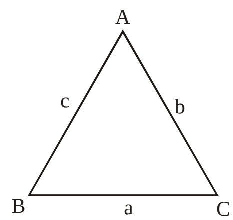
5 INTRODUCTION TO TRIGNOMETRY

5.1 INTRODUCTION

The word ‘trignometry’ is comes from greek word. ‘tri’ (means three) gon (means side) and metron (means measurment). In lavel trignometry is the study of relationship between the side and angle of triangle.

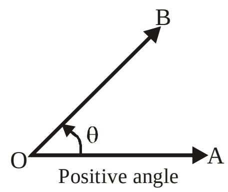


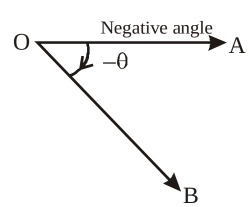
5.2 MEASUREMENT OF ANGLES

Thus, an angle is considered as the figure obtained by rotating a given ray about its end-point.

Measure of an angle: The measure of an angle is the amount of rotation from the initial side to the terminal side.

Sence of An angle: The sense of an angle is said to be positive or negative according as the initial side rotates in anticlockwise direction to the terminal side.





5.3 SYSTEMS OF MEASUREMENT OF ANGLE

5.3.1 Sexagesimal system

In this system a right angle is divided into 90 equal parts, called degree. The symbol 1° is used to denote one degree. Thus, one degree is one-ninetieth part of a right angle. Each degree is divided into 60 equal parts, called minutes. The symbol 1' is used to denote one minute. And each minute is divided into 60 equal parts, called second. The symbol 1'' is used to denote one second.

Thus, 1 right angle = 90 degrees (= 90°)

1° = 60 mintues (= 60')

1' = 60 seconds (= 60")

5.3.2 Centesimal system

In this system a right angle is divided into 100 equal parts, called grades; each grade is subdivided into 100 minutes, and each minute into 100 seconds.

The symbols 1^g, $1'$ and $1''$ are used to denotes a grade, a minute, and a second respectively.

Thus, 1 right angle = 100 grades ($= 100^g$)

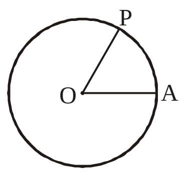
1 grade = 100 minutes (=100')

1 minute = 100 seconds (= 100")

5.3.3 Circular System

In this system the unit of measurement is radian as defined below.

One radian, written as 1^c, is the measure of an angle subtended at the centre of a circle an arc of length equal to the radius of the circle.



Consider a circle of radius r having centre at O. Let A be a point on the circle. Now cut off an arc AP whose length is equal to the radius r of the circle. They by the definition of the measure ∠AOP is 1 radian ($= 1^c$).

5.4 RELATION BETWEEN DEGREES AND RADIANS

Consider a circle with centre O and radius r. Let A be a point on the circle. Join OA and cut off an arc OP of length equal to the radius of the circle.Then, ∠AOP = 1 radian. Produce AO to meet the circle at B.

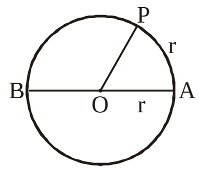
$\therefore$ ∠AOB = a straight angle = 2 right angles

We know that the angles at the centre of a circle are propertional to the arcs subtending them.

$$\therefore \frac{\angle AOP}{\angle AOB} = \frac{\text{arc AP}}{\text{arc APB}}$$

$$\Rightarrow \frac{\angle AOP}{2 \text{ right angles}} = \frac{r}{\pi r}$$

$$\Rightarrow \angle AOP = \frac{2 \text{ right angles}}{\pi}$$



$$\Rightarrow 1^c = \frac{180°}{\pi}$$

Hence, one radian = $\frac{180°}{\pi}$ $\Rightarrow \pi$ radians = 180°

Remark 1

When an angle is expressed in radians, the word radian is generally omitted.

Remark 2

Since 180° = π radians. Therefore, 1° = π/180 radian.

Hence, $30° = \frac{\pi}{180} \times 30 = \frac{\pi}{6}$ radians,

$45° = \frac{\pi}{180} \times 45 = \frac{\pi}{4}$ radians,

$60° = \frac{\pi}{180} \times 60 = \frac{\pi}{3}$ radians,

$90° = \frac{\pi}{180} \times 90 = \frac{\pi}{2}$ radians,

Remark 3

We have,

π radians = 180°

$\therefore$ 1 radian = $\frac{180°}{\pi} = \left(\frac{180}{22} \times 7\right)^{o}$ 57°16'22''(approx).

Remark 4

We have,

180° = π radians

$\Rightarrow 1° = \frac{\pi}{180}$ radian $= \left(\frac{22}{7 \times 180}\right)$ radian = 0.01746 radian.

5.5 RELATION BETWEEN THREE SYSTEMS OF MEASUREMENT OF AN ANGLE

Let D be the number of degrees, R be the number of radians and G be the number of grades in an angle θ.

Now, $90° = 1$ right angle

$\Rightarrow 1° = \frac{1}{90}$ right angle

$\Rightarrow D° = \frac{D}{90}$ right angles

$\Rightarrow \theta = \frac{D}{90}$ right angles

Angles, π radians = 2 right angles

$\Rightarrow$ 1 radian = $\frac{2}{\pi}$ right angles

$\Rightarrow$ R radians = $\frac{2R}{\pi}$ right angles

Add, 100 grades = 1 right angle $\Rightarrow$ 1 grade = $\frac{1}{100}$ right angle

$\Rightarrow$ G grades = $\frac{G}{100}$ right angles $\Rightarrow \theta = \frac{G}{100}$ right angles

From (i), (ii) and (iii), we get

$$\frac{D}{90} = \frac{G}{100} = \frac{2R}{\pi}$$

This is required relation between the three systems of measurement of an angle.

Illustration 1

Find the degree measure corresponding to the following radian measures.

(i) $\left(\frac{2\pi}{15}\right)^c$ (ii) -2^c

Illustration 2

Find the radian measures corresponding to the following degree measures:

(i) $-37°30'$ (ii) $5°\,37'\,30''$ (iii)

Illustration 3

Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring 15°.

Solution

Let s be the length of the arc subtending an angle θ^c at the centre of a circle or radius r. Then, $\theta = \frac{s}{r}$.

Here, r = 5 cm and θ = $15° = \left(15 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{12}\right)^c$

$$\therefore \theta = \frac{s}{r} \Rightarrow \frac{\pi}{12} = \frac{s}{4} \Rightarrow s = \frac{5\pi}{12} \text{ cm.}$$

Illustration 4

Find the degrees the angle subtended at the centre of a circle of diameter 50 cm by an arc of length 11 cm.

Solution

Here, r = 25 cm and s = 11 cm

$$\therefore \theta = \left(\frac{s}{r}\right)^c \Rightarrow \theta = \left(\frac{11}{25}\right)^c = \left(\frac{11}{25} \times \frac{180°}{\pi}\right)^o = \left(\frac{11}{25} \times \frac{180}{22} \times 7\right)^o$$

$$\Rightarrow \theta = \left(\frac{126}{5}\right)^o = \left(25\frac{1}{5}\right)^o = 25°\left(\frac{1}{5} \times 60\right)' = 25°12'$$

Illustration 5

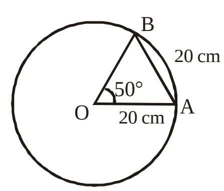
In a circle of diameter 40 cm the length of a chord is 20 cm. Find the length of minor arc corresponding to the chord.

Solution

Let arc AB = s. It is given that OA = 20 cm and chord AB = 20 cm. Therefore, ΔOAB is an equilateral triangle. Hence,

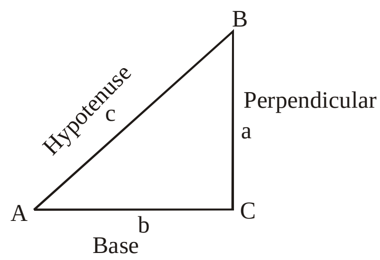
$$\angle AOB = 60° = \left(60 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{3}\right)^c$$

Now, $\theta = \frac{\text{arc}}{\text{radius}} \Rightarrow \frac{\pi}{3} = \frac{s}{20} \Rightarrow s = \frac{20\pi}{3}$ cm.



5.6 TRIGNOMETRIC RATIO

With reference to angle A in a right angled Δ ABC, right angle at C.



a is opposite side of angle ∠A (Perpendicular), b is opposite side of angle ∠B c is opposite side of angle ∠C.

The ratio of sides $\frac{a}{c}, \frac{b}{c}, \frac{a}{b}, \frac{b}{a}, \frac{c}{b}, \frac{c}{a}$ have the following names

$\frac{a}{c}$ is called the sine of A, written as sin A

$\frac{b}{c}$ is called the co-sine of A, written as cos A

$\frac{a}{b}$ is called the tangent of A, written as tan A.

$\frac{b}{a}$ is called the co-tangent of A. written as cot A

$\frac{c}{b}$ is called the secant of A, written as sec A

$\frac{c}{a}$ is called the co-secant of A, written as cosec A.

So there are six trignometric Ratio

$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{a}{c} \Rightarrow \cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{b}{c}$

$\tan A = \frac{\text{Prependicular}}{\text{Base}} = \frac{a}{b} \Rightarrow \cot A = \frac{\text{Base}}{\text{Prependicular}} = \frac{b}{a}$

$\text{cosec } A = \frac{\text{Hypotenuse}}{\text{Prependicular}} = \frac{c}{a} \Rightarrow \sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{c}{b}$

Fundamental Relation

(a) Reciprocal Relation

cosec A = $\frac{1}{\text{Sin A}}$, sin A = $\frac{1}{\text{Cosec A}}$

sec A = $\frac{1}{\text{Cot A}}$, cos A = $\frac{1}{\text{sec A}}$

cot A = $\frac{1}{\text{tan A}}$, tan A = $\frac{1}{\text{cot A}}$

(b) Quotient Relation

tan A = $\frac{\text{Sin A}}{\text{cos A}}$ cot A = $\frac{\text{cos A}}{\text{sin A}}$

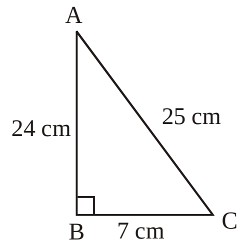
Illustration 6

Find trignometric ratio:

In a triangle ABC, right-angled at B, AB = 24 cm, BC = 7 cm. Determine

(i) sin A, Cos A

(ii) Sin C, Cos C



Solution

In Δ ABC

$AB^2 + BC^2 = AC^2$

$(24)^2 + (7)^2 = AC^2$

$AC^2 = 625$

AC = 25

hypotenuse = 25 cm

(i) For ∠A, AB = base

Perpendicular = BC &

hypotenuse = AC

sin A = $\frac{\text{Perpendicular}}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{7}{25}$

Cos A = $\frac{\text{Base}}{\text{hypotenuse}} = \frac{24}{25}$

(ii) For $\angle C$ base = BC and

Perpendicular = AB and

hypotenuse = AC

$$\sin C = \frac{\text{Perpendicular}}{\text{hypotenuse}} = \frac{24}{25}$$

$$\text{Cos } C = \frac{\text{Base}}{\text{hypotenuse}} = \frac{7}{25}$$

$\therefore$ **(B)**

Angle / Ratio	0º	30º	45º	60º	90º
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cot	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cosec	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Illustration 7

Find the value of the following:

(i) $4\cos^2 60^o + 4\sin^2 45^o - \sin^2 30^o$

Solution

(i) $4\cos^2 60^o + 4\sin^2 45^o - \sin^2 30^o$

$$= 4\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$= 4 \times \frac{1}{4} + 4 \times \frac{1}{2} - \frac{1}{4}$$

$$= \frac{11}{4}$$

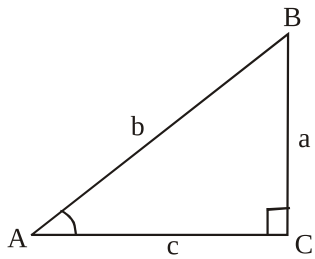
5.7 TRIGNOMETRIC RATIO OF COMPLEMENTRY ANGLES

Let Δ ABC be a right angle triangle at $\angle C = 90^\circ$

so $\angle A + \angle B = 90^\circ$

So $\angle A$ & $\angle B$ are complement angle of each other

$\angle B = 90^\circ - \angle A$



$$\sin A = \frac{a}{c} = \cos B = \cos (90^\circ - A)$$

$$\cos A = \frac{b}{c} = \sin B = \sin (90^\circ - A)$$

$$\tan A = \frac{a}{b} = \cot B = \cot (90^\circ - A)$$

$$\cot A = \frac{b}{a} = \tan B = \tan (90^\circ - A)$$

$$\sec A = \frac{c}{b} = \text{cosec } B = \text{cosec } (90^\circ - A)$$

$$\text{cosec } A = \frac{c}{a} = \sec B = \sec (90^\circ - A)$$

so $\sin (90^\circ - A) = \cos A$ $\qquad$ $\cos (90^\circ - A) = \sin A$

$\tan (90^\circ - A) = \cot A$ $\qquad$ $\cot (90^\circ - A) = \tan A$

$\sec (90^\circ - A) = \text{cosec } A$ $\qquad$ $\text{cosec } (90^\circ - A) = \sec A$

Illustration 8

$\sin 30^\circ = \sin (90^\circ - 60^\circ) = \cos 60^\circ$

$\tan 68^\circ = \tan (90^\circ - 22^\circ) = \cot 22^\circ$

Illustration 9

With out using trignometric tables, find the value of

(a) $\sin^2 40^\circ - \cos^2 50^\circ$ $\qquad$ ***(b)*** $\dfrac{\cot 54^\circ}{\tan 36^\circ} + \dfrac{\tan 20^\circ}{\cot 70^\circ} - 2$

Solution

(a) $\sin^2 40^\circ - \cos^2 (90^\circ - 40^\circ)$

$= \sin^2 40^\circ - \sin^2 40^\circ$

$= 0$

(b) $\dfrac{\cot 54^\circ}{\tan 36^\circ} + \dfrac{\tan 20^\circ}{\cot 70^\circ} - 2$

$= \dfrac{\cot(90^\circ - 36^\circ)}{\tan 36^\circ} + \dfrac{\tan 20^\circ}{\cot(90^\circ - 20^\circ)} - 2$

$= \dfrac{\tan 36^\circ}{\tan 36^\circ} + \dfrac{\tan 20^\circ}{\tan 20^\circ} - 2$

$= 1 + 1 - 2 = 0$

5.8 TRIGNOMETRIC IDENTITIES

If two expressions are equal for all values of the variables, then the relation is called an identities $\dfrac{x+2}{3} + \dfrac{x+3}{2} = \dfrac{5x+13}{6}$ is an identity since L.H.S. = R.H.S for all real value of x.

There are three fundamental trignometric identities

(i) $\cos^2 A + \sin^2 A = 1$

(ii) $1 + \tan^2 A = \sec^2 A$

(iii) $\cot^2 A + 1 = \text{cosec}^2 A$

(i) $\cos^2 A + \sin^2 A = 1$

Solved Examples

Example 1

If θ is an acute angle and $\tan\theta + \cot\theta = 2$, find the value of $\tan^7\theta + \cot^7\theta$.

Solution

$\tan\theta + \cot\theta = 2$

$\Rightarrow \tan\theta + \dfrac{1}{\tan\theta} = 2$

$\Rightarrow \tan^2\theta + 1 = 2\tan\theta$

$\Rightarrow \tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow (\tan\theta - 1)^2 = 0$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \theta = 45^\circ$

$\tan^7\theta + \cot^7\theta = (\tan 45^\circ)^7 + (\cot 45^\circ)^7$

$= 1 + 1 = 2$

Example 2

If $\dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \dfrac{1-\sqrt{3}}{1+\sqrt{3}}$ then find an acute angle θ.

Solution

$$\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\frac{1-\sqrt{3}}{1+\sqrt{3}}$$

Applying componendo and devidendo

$$\Rightarrow \quad \frac{(\cos\theta-\sin\theta)+(\cos\theta+\sin\theta)}{(\cos\theta-\sin\theta)-(\cos\theta+\sin\theta)}=\frac{\left(1-\sqrt{3}\right)+\left(1+\sqrt{3}\right)}{\left(1-\sqrt{3}\right)-\left(1+\sqrt{3}\right)}$$

$\Rightarrow \quad \frac{2\cos\theta}{-2\sin\theta}=\frac{2}{-2\sqrt{3}}$ $\quad\Rightarrow \quad \cot\theta=\frac{2}{\sqrt{3}}$

$\Rightarrow \quad \tan\theta=\sqrt{3}$ $\quad\Rightarrow \quad \theta = 60°$

Example 3

In an acute angled triangle ABC if tan(A+B–C) = 1 and sec(B+C–A) = 2, find the value of A,B and C.

Solution

We have

$\Rightarrow$ tan(A + B–C) = 1 and sec(B+C–A) = 2

$\Rightarrow$ tan(A + B–C) = tan 45° and sec(B+C–A) = sec60°

$\Rightarrow$ A + B – C = 45° and B + C –A = 60° $\quad\Rightarrow$ 2B = 105°

$\Rightarrow$ B= $52\frac{1}{2}°$

Putting $52\frac{1}{2}°$ in B + C –A = 60°, we get

$52\frac{1}{2}°$ + C – A = 60° $\quad\Rightarrow$ C –A = $7\frac{1}{2}°$

Also, in ΔABC we have

A + B + C = 180° $\quad\Rightarrow$ A + $52\frac{1}{2}°$ + C = 180°

$\Rightarrow$ C + A = $127\frac{1}{2}°$

Adding and substituting (i) and (ii), we get

2C = 135° and 2A = 120° $\quad\Rightarrow$ C = $67\frac{1}{2}°$ and A = 60°

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So, A = 60°, B = $52\frac{1}{2}°$ and C = $67\frac{1}{2}°$

Example 4

Prove that

(a) $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \tan\theta + \cot\theta$

(b) $\frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} + 1} = \frac{1 + \cos A}{\sin A}$

Solution

(a) LHS $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}$

$= \sqrt{1 + (\tan^2\theta) + (1 + \cot^2\theta)}$

$= \sqrt{2 + \tan^2\theta + \cot^2\theta}$

$= \sqrt{\tan^2\theta + 2\tan\theta.\cot\theta + \cot^2\theta}$

$= \sqrt{(\tan\theta + \cot\theta)^2}$ = tanθ + cotθ RHS

(b) LHS $\frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1}$

$= \frac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec}^2 A - \cot^2 A)}{(\cot A - \operatorname{cosec} A) + 1}$

$= \frac{(\operatorname{cosec} A + \cot A) - (\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot A)}{\cot A - \operatorname{cosec} A + 1}$

$= \frac{(\operatorname{cosec} A + \cot A) \times [1 - (\operatorname{cosec} A - \cot A)]}{(\cot A - \operatorname{cosec} A + 1)}$

$= \frac{(\operatorname{cosec} A + \cot A) - [1 - \operatorname{cosec} A + \cot A]}{(1 - \operatorname{cosec} A + \cot A)}$

= cosecA + cotA RHS

Example 5

If a$\sec\theta$ ***+ b***$\tan\theta$ ***+ c = 0 and p***$\sec\theta$ ***+ q***$\tan\theta$ ***+ r = 0. Prove that*** $(br-qc)^2 - (pc-ar)^2 = (aq-pb)^2$

Solution

We have

asecθ +btanθ + c = 0

and psecθ + qtanθ + r = 0

Solving these two equation for secθ and tanθ by the cross multiplication, we get

$$\frac{\sec\theta}{br - qc} = \frac{\tan\theta}{cp - ar} = \frac{1}{aq - bp}$$

$\sec\theta = \dfrac{br - cq}{aq - bp}$ and $\tan\theta = \dfrac{cp - ar}{aq - bp}$

Now, $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \left(\dfrac{br - cq}{aq - bp}\right)^2 - \left(\dfrac{cp - ar}{aq - bp}\right)^2 = 1$

$\Rightarrow (br - cq)^2 - (cp - ar)^2 = (aq - bp)^2$

Example 6

If $\sin\theta + \sin^2\theta + \sin^3\theta = 1$, then prove that $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$

Solution

We have

$\sin\theta + \sin^2\theta + \sin^3\theta = 1$

$\Rightarrow \sin\theta + \sin^3\theta = 1 - \sin^2\theta$ $\Rightarrow \sin\theta\,(1 + \sin^2\theta) = \cos^2\theta$

Now squaring both side

$\Rightarrow \sin^2\theta\,(1 + \sin^2\theta)^2 = \cos^4\theta$

$\Rightarrow (1 - \cos^2\theta)\,\{1 + (1 - \cos^2\theta\}^2 = \cos^4\theta$

$\Rightarrow (1 - \cos^2\theta)\,\{2 - \cos^2\theta\}^2 = \cos^4\theta$

$\Rightarrow (1 - \cos^2\theta)\,(4 - 4\cos^2\theta + \cos^4\theta) = \cos^4\theta$

$\Rightarrow 4 - 4\cos^2\theta + \cos^4\theta - 8\cos^2\theta + 4 = 0$

$\Rightarrow \cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$

Example 7

If $\sec\theta + \tan\theta = p$, obtain the value of $\sec\theta$, $\tan\theta$ and $\sin\theta$ in terms of P

Solution

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)\;(\sec\theta - \tan\theta) = 1$ $\Rightarrow p(\sec\theta - \tan\theta) = 1$

$\Rightarrow (\sec\theta - \tan\theta) = 1/p$

$\sec\theta + \tan\theta = p$

$\sec\theta - \tan\theta = 1/p$

———————————————

$2\sec\theta = p + \dfrac{1}{p} = \dfrac{p^2 + 1}{p}$

$\sec\theta = \dfrac{p^2 + 1}{2p}$ and $\tan\theta = \dfrac{p^2 - 1}{2p}$

$\sin\theta = \dfrac{\tan\theta}{\sec\theta} = \dfrac{p^2 - 1}{p^2 + 1}$

EXERCISE-I

1. If cosec θ = $\frac{13}{12}$, find the value of $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$

2. Without using trignometrical tables, find the vlaue of

(a) $\frac{\cos 35^o}{\sin 55^o} + \frac{\tan 27^o . \tan 53^o}{\sin 30^o} - 3\tan^2 60^o$.

(b) $\frac{\sin 39^o}{\cos 51^o} + 2\tan 11^o . \tan 31^o . \tan 45^o . \tan 59^o . \tan 79^o - 3(\sin^2 21^o + \sin^2 69^o)$

3. Prove the following identities

(a) $\frac{1}{\sec x - \tan x} - \frac{1}{\cos x} = \frac{1}{\cos x} - \frac{1}{\sec x + \tan x}$

(b) $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\text{cosec}\theta$

(c) $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = 2\sec\theta$

(d) $\frac{1+\sec\theta}{\sec\theta} = \frac{\sin^2\theta}{1-\cos\theta}$

(e) $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \sin A + \cos A$

(f) $(\cot\theta - \text{cosec}\,\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

4. Solve the following equation for $0^o \le \theta \le 90^o$

(a) $2\sin 2\theta = \sqrt{3}$ (b) $2\cos^2\theta - \cos\theta = 0$ (c) $3\sec^2\theta = 2\text{cosec}\,\theta$

(d) $\frac{\cos\theta}{\text{cosec}\theta + 1} + \frac{\cos\theta}{\text{cosec}\theta - 1} = 2\tan\theta$ (e) $2\sin^2\theta - 3\sin\theta + 1 = 0$

5. If sin (A – B) = sin A . cos B – cos A . sin B, prove that the value of $\sin 15^o = \frac{\sqrt{3}-1}{2\sqrt{2}}$.

6. If cos (A + B) = cos A . cos B. – sin A . sin B, prove that the value of $\cos 75^o = \frac{\sqrt{3}-1}{2\sqrt{2}}$.

7. If $\cos(A + B) = \frac{1}{2}$, & $\sin(A - B) = \frac{1}{2}$, $0 < A + B \leq 90^o$, $A > B$, find A and B.

8. If $\sin(A + B) = 1$ & $\sin(A - B) = \frac{1}{2}$, $0 < A + B \leq 90^o$, $A > B$, find A and B.

9. $\tan(A-B) = \frac{1}{\sqrt{3}}$ and $\tan(A+B) = \sqrt{3}$, $0° < A+B \leq 90°$, $A > B$. Find A and B.

10. $\frac{\sec^2 54° - \cot^2 36°}{\operatorname{cosec}^2 57° - \tan^2 33°} + 2\sin^2 38° \sec^2 52° - \sin^2 45°$

11. The angles of a triangle are in A.P. The number of degrees in the least is to the number of radians in the greatest as $60 : \pi$. Find the angles in degrees.

12. The angles of a triangle are in A.P. The number of grades in the least, is to the number of radians in the greatest as $40 : \pi$. Find the angles in degrees.

13. Expresse the angular measurement of the angle of a regular decagon in degrees, grade and radians.

14. If the arcs of same length in two circles subtend angles of 60° and 75° at their centres. Find the ratio of thier radii.

15. find in degrees the angle through which a pendulum swings of its length is 50 cm and the tip desceibes an arc of length 10 cm.

16. A horse is tied to a post by a rope. If the horse moves along a circular path always keeping the rope tight and describes 88 metres when it has traced out 72° at the centre, find the length of the rope.

17. If the angular diameter of the moon be 30', how far from the eye a coin of diameter 2.2 cm be kept to hide the moon?

18. Assuming that a person of normal sight can read print at such a distance that the letters subtend an angle of 5' at his eye, find what is the height of the letters that he can read at a distance of 12 metres.

19. Find the angle between the minute hand of a clock and the hour hand when the time is 7:20 AM.

20. Find in degrees and radians the angle between the hour hand and the minute-hand of a clock at half past three.

EXERCISE-II

1. If $n \tan\theta = m$ evaluate $\frac{m\sin\theta + n\cos\theta}{m\sin\theta - n\cos\theta}$.

2. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then find $\sin\theta \times \cos\theta$.

3. Find the value of

(a) $\frac{\sin 30^o}{\cos 45^o} + \frac{\tan 45^o}{\sec 60^o} - \frac{\sin 60^o}{\cot 45^o} - \frac{\cos 30^o}{\sin 90^o}$

(b) $\frac{\tan 45^o}{\operatorname{cosec} 30^o} + \frac{\sec 45^o}{\cot 45^o} - \frac{2\sin 90^o}{\cos 0^o}$

4. If $x = a\cos\theta - b\sin\theta$, $y = a\sin\theta + b\cos\theta$. Prove that $x^2 + y^2 = a^2 + b^2$.
5. If $a\sin^3\theta + b\cos^3\theta = \sin\theta \cdot \cos\theta$ and $a\sin\theta - b\cos\theta = 0$, then prove that $a^2 + b^2 = 1$.
6. If $x = r\sin\theta \cdot \cos\theta$, $y = r\sin\theta \cdot \sin\theta$ & $z = r\cos\theta$ then prove that $r^2 = x^2 + y^2 + z^2$.
7. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$ then prove that $m^2 - n^2 = 4\sqrt{mn}$
8. If $\cos\theta + \sin\theta = m$ and $\sec\theta + \operatorname{cosec}\theta = n$ then prove that $n(m^2 - 1) = 2m$
9. If $\sec\theta = x + \frac{1}{4x}$, then prove that $\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$
10. If $x = a\sec\theta \cdot \cos\phi$, $y = b\sec\theta \cdot \sin\phi$ and $z = c\tan\theta$ show that $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$.
11. Prove that: $\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots\ldots\ldots\ldots \tan 89^\circ = 1$
12. $(a^2 - b^2)\sin\theta + 2ab\cos\theta = a^2 + b^2$, find the value of $\tan\theta$.

EXERCISE-III

SECTION-A

- **Multiple choice question with one correct answers**

1. If $\sin\theta + \cos\theta = \sqrt{2}\cos(90^\circ - \theta)$ then $\cot\theta$ is equal to

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{\sqrt{2}-1}$ (D) $\sqrt{2}-1$

2. If $\sec\theta + \tan\theta = x$ then the value of $\sec\theta - \tan\theta$ is equal to

(A) $-x$ (B) $\frac{1}{x}$ (C) $-\frac{1}{x}$ (D) $\sqrt{x}$

3. If $x = a\sin\theta$ and $y = b\cos\theta$, then the value of $b^2x^2 + a^2y^2$ is

(A) a^2b^2 (B) ab (C) $\frac{1}{a^2b^2}$ (D) $\frac{1}{ab}$

4. An equation is called an identity if
   (A) If is true for all values of variable
   (B) Not for all values of variabls but some value of variable
   (C) Exactly one value of variables
   (D) Exactly two value of variables
5. If $x = (\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$ & $y = (\sec A - \tan A)(\sec B + \tan B)(\sec C + \tan C)$ and $x = y$ then x & y is equal to

(A) ± 1 (B) 0 (C) ± 2 (D) None of these

6. If $x = \cot^2\theta - \frac{1}{\sin^2\theta}$ than the value of x is

(A) 1 (B) – 1 (C) ± 1 (D) zero

7. $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta)$ is equal

(A) zero (B) 1 (C) – 1 (D) None of these

8. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ is equal to

(A) $\sec\theta + \tan\theta$ (B) $\sec\theta - \tan\theta$ (C) $\sec^2\theta + \tan^2\theta$ (D) $\sec^2\theta - \tan^2\theta$

9. $\sec^4 A - \sec^2 A$ is equal to

(A) $\tan^2 A - \tan^4 A$ (B) $\tan^4 A - \tan^2 A$ (C) $\tan^4 A + \tan^2 A$ (D) $\tan^2 A + \tan^4 A$

10. $\cos^4 A - \sin^4 A$ is equal to

(A) $2\cos^2 A + 1$ (B) $2\cos^2 A - 1$ (C) $2\sin^2 - 1$ (D) $2\sin^2 A + 1$

11. $P = (1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta)$

the value of P is equal to

(A) 1 (B) 2 (C) 4 (D) zero

12. $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$ is equal to

(A) zero (B) 1 (C) – 1 (D) none of these

13. The value of $\dfrac{\dfrac{2\sin(140°)\sec(280°)}{\sec(220°)} + \dfrac{\sec(340°)}{\cos(20°)}}{\dfrac{\cot 200° - \tan(280°)}{\cot(200°)}}$ is equal to:

(A) $\sqrt{3}$ (B) $-\sqrt{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) $-\frac{1}{\sqrt{3}}$

14. The expression $\sqrt{\sin^4(37.5)° + 4\cos^2(37.5)°} + \sqrt{\cos^4(37.5)° + 45m^2(37.5)°}$ simplifies to:

(A) an irrational number (B) a prime number

(C) a natural number which is not composite (D) a real number of the form $a + \sqrt{b}$

15. If $15\sin^4\alpha + 10\cos^4\alpha = 6,$ then the value of $8\operatorname{cosec}^4\alpha + 27\sec^6\alpha$ is

(A) 200 (B) 250 (C) 220 (D) None of these

16. If $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 - bx + c = 0$, then

(A) $a^2 - b^2 = 2ac$ (B) $a^2 + b^2 = 2ac$ (C) $a^2 + b^2 + 2ac = 0$ (D) $b^2 - a^2 = 2ac$

17. If $0 < \theta < \pi$ $2\sin^2\theta + 5\sin\theta - 3 = 0$, then θ, in radians must be

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{5\pi}{6}$

18. Exact value of $\cos^2 73° + \cos^2 47° - \sin^2 43° + \sin^2 107°$ is equal to:

(A) ½ (B) ¾ (C) 1 (D) None of these

19. If $\sin\theta$ and $\sec\theta$ $(0 < \theta < \pi/2)$ are the roots of the equation $2x^2 + kx + 1 = 0$, then the value of 'k' is equal to

(A) $-\frac{7\sqrt{2}}{5}$ (B) $\frac{7\sqrt{5}}{5}$ (C) $\frac{7\sqrt{5}}{2}$ (D) $-\frac{7\sqrt{5}}{5}$

SECTION-B

- **Match the following (one to one)**

Column-I and **column-II** contains **four** entries each. Entries of column-I are to be matched with some entries of column-II. Only One entries of column-I may have the matching with the some entries of column-II and one entry of column-II Only one matching with entries of column-I

1.

Column I	**Column II**
(A) If $\tan^2\theta + \cot^4\theta = x$ then the value of x is	(P) $x \le 1/4$
(B) If $\sin^4\theta - \cos^4\theta = x$ then the value of x is	(Q) $3/4 \le x \le 1$
(C) If $\sin^4\theta.\cos^4\theta = x$ then the value of x is	(R) $x \le 1$
(D) If $\cos^4\theta + \sin^4\theta = x$ then the value of x is	(S) $x \ge 2$

EXERCISE-IV

SECTION-A

- **Multiple choice question with one correct answers**

1. If $a = \dfrac{\cot\theta}{\cot\theta - \cot 3\theta}$ & $b = \dfrac{\tan\theta}{\tan\theta - \tan 3\theta}$ then $\sqrt{a+b}$ is equal to

(A) ± 2 (B) -2 (C) $+1$ (D) -1

2. If $a\cos\theta - b\sin\theta = C$ then $a\sin\theta + b\cos\theta =$

(A) $\pm\sqrt{a^2+b^2+c^2}$ (B) $\pm\sqrt{a^2+b^2-c^2}$ (C) $\pm\sqrt{c^2-a^2-b^2}$ (D) None of these

3. If $a\cos\theta + b\sin\theta = 4$ and $a\sin\theta - b\cos\theta = 3$ then $(a^2 + b^2)$ is equal to

(A) 7 (B) 12 (C) 25 (D) None of these

4. If $\sin\theta + \sin^2\theta + \sin^3\theta = 1$ the value of $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = p$ the value of p is
(A) 1 (B) 2 (C) 3 (D) 4

5. If $\cos(\theta + \phi) = m\cos(\theta - \phi)$ then $\tan\theta$ is equal to
(A) $\left(\frac{1+m}{1-m}\right)\tan\phi$ (B) $\left(\frac{1-m}{1+m}\right)\tan\phi$ (C) $\left(\frac{1-m}{1+m}\right)\cot\phi$ (D) $\left(\frac{1+m}{1-m}\right)\cot\phi$

6. If $\sin\theta - \cos\theta = 1$ then the value of $\sin^3\theta - \cos^3\theta$ is if ($\theta \in R$)
(A) zero (B) 1 (C) 2/3 (D) 3/2

7. If $0° \le \theta \le 90°$ and $\sqrt{3}\tan\theta - \sec\theta = 1$ then θ has the value
(A) 30° (B) 45° (C) 60° (D) 90°

8. If $a\cot\theta + b\operatorname{cosec}\theta = P$ and $b\cot\theta + a\operatorname{cosec}\theta = q$, then $p^2 - q^2$ is equal to:
(A) $a^2 - b^2$ (B) $b^2 - a^2$ (C) $a^2 + b^2$ (D) b - a

9. If $\sin^4\theta - \cos^4\theta = x$ then the value of x is
(A) $2\sin^2\theta - 1$ (B) $1 - 2\sin^2\theta$ (C) $\cos^2\theta - \sin^2\theta$ (D) $\sin^2\theta + \cos^2\theta$

10. If $\tan^2\theta + \cot^2\theta = 17/4$ then $(\tan\theta + \cot\theta)$ is equal to
(A) $\frac{\sqrt{17}}{2}$ (B) 5/2 (C) $\frac{\sqrt{17}}{2} + \sqrt{2}$ (D) $\frac{\sqrt{17}}{2} - \sqrt{2}$

SECTION-B

- **Multiple choice question with one or more than one correct answers**

1. Given that θ lies in the first quadrant and $\cos\theta = \tan\theta$ then $\sin\theta$ is equal to
(A) $\frac{-1+\sqrt{5}}{2}$ (B) $\frac{1+\sqrt{5}}{2}$ (C) $\frac{-1-\sqrt{5}}{2}$ (D) $-\left(\frac{1+\sqrt{5}}{2}\right)$

2. If $y = \tan\alpha\sqrt{1-\sin^2\alpha}$ the y is equal to
(A) $\sin\alpha$ (B) $\cos\alpha$ (C) $\sqrt{1-\cos^2\alpha}$ (D) $\sqrt{1-\sin^2\alpha}$

3. Value of $\frac{\sin 60° + \cos 60°}{\tan 60°}$ is
(A) $\frac{2\sqrt{3}}{\sqrt{3}+1}$ (B) $\frac{\sqrt{3}+1}{2\sqrt{3}}$ (C) $\frac{1}{\sqrt{3}(\sqrt{3}-1)}$ (D) $\sqrt{3}(\sqrt{3}-1)$

SECTION-C

- **Comprehension**

If $p = (\sec A + \tan A)(1 - \sin A)$, $q = \frac{1+\tan^2 A}{1+\cot^2 A}$ then

1. The value of p^2q is
(A) $\sec^2 A$ (B) $\sin^2 A$ (C) $1 - \cos^2 A$ (D) $\cos^2$