8 QUADRATIC EQUATION

8.1 INTRODUCTION

A polynomial of degree one is called linear polynomial and that of degree two is called quadratic polynomial for example $x^2 + 4$, $2x^2 + 3x + 4$, are quadratic polynomial.

A quadratic polynomial can have at most three terms namely the terms containing x^2 , x and the constant term.

The gereral form of a quadratic polynomial is x in $ax^2 + bx + c$, where a,b,c are real numbers and $a \neq 0$. When we equate this polynomial to zero we get a quadratic equation.

8.2 QUADRATIC EQUATION

(i) Let $p(x) = ax^2 + bx + c$, where $a, b, c \in R$ be the quadratic polynomial, then $p(x) = 0$, i.e. $ax^2 + bx + c = 0$ is called a **quadratic equation** where a, b, c are real numbers and $a \neq 0$

Since $a \neq 0$, quadratic equations in general are of the following types

(a) $b = 0, c \neq 0$ ie $ax^2 + c = 0$

(b) $b \neq 0, c = 0$ ie $ax^2 + bx = 0$

(c) $b = 0, c = 0$ ie $ax^2 = 0$

(d) $b \neq 0, c \neq 0$ ie $ax^2 + bx + c = 0$

(ii) An equation involving the square of unknown quantity(variable) and no other higher power, is called a quadratic equation or a second degree equation.

Illstration 1

Which of the following are quadratic equations.

(i) $3x^2 - 8x = 0$ *(ii)* $x^2 + \frac{1}{x^2} = 8$

(iii) $x^2 - 6x + 5\sqrt{x} - 7 = 0$

Ans

(i)

8.3 METHODS OF SOLVING QUADRATIC EQUATIONS

Method: 1

8.3.1 Solution of quadratic equations By factorisation method:

Example:

Solve: $x^2 - 7x = -10;$

$\Rightarrow x^2 - 7x + 10 = 0$

$\Rightarrow (x - 2)(x - 5) = 0$

$\Rightarrow x = 2$ and $x = 5$

Hence the roots of equation are $x = 2$ and $x = 5$

Method: 2

8.3.2 Solution of quadratic equations by completion of square:

Illustration showing solution of a quadratic equation using completion of square:

Find the roots of $2x^2 + 6x + 1 = 0$

Solution

Here the coefficient of x^2 is not equal to 1 so first we convert the coefficient of x^2.

Hence the given equation can be written as : $2(x^2 + 3x + 1/2) = 0$ (Dividing both sides of the equation by 2)

Now coefficient of x is 3, so adding and subtracting $\left(\frac{3}{2}\right)^2$ to the given equation:

$$2\left[x^2 + 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + \frac{1}{2}\right] = 0 \quad \Rightarrow \quad 2\left[\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + \frac{1}{2}\right] = 0$$

$$2\left[\left(x + \frac{3}{2}\right)^2 - \frac{7}{4}\right] = 0 \quad \Rightarrow \quad \left(x + \frac{3}{2}\right)^2 = \frac{7}{4}$$

$$x + \frac{3}{2} = \pm\frac{\sqrt{7}}{2} \quad \Rightarrow \quad x = -\frac{3}{2} \pm \frac{\sqrt{7}}{2}$$

Hence, $x = \dfrac{-3 + \sqrt{7}}{2}$, $x = \dfrac{-3 - \sqrt{7}}{2}$

Steps used to solve quadratic equation

(i) Make the coefficient of x^2 is 1 (if any)

(ii) Add and subtract the square of half of the coefficient of x and simplify

(iii) In this way we obtain an expression of the form $A^2 - B^2 = 0$ which gives $A^2 - B^2 = (A + B)(A - B) = 0$.

(iv) Equate each factor is equal to zero. The values of x so obtained are required zeros of the given quadratic equation.

Method: 3

8.3.3 Solving of quadratic equations by the formula method (Shridharacharya method)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So roots are

$$\mathbf{x} = \frac{-\mathbf{b} + \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}} ; \frac{-\mathbf{b} - \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}$$

8.4 DISCRIMINANT AND NATURE OF ROOTS

(i) Discriminant

For the quadratic equation $ax^2 + bx + c = 0$, the expression $b^2 - 4ac$ denoted by '**D**' is called **DISCRIMINENT** of the equation.

(ii) Nature of the roots:

Nature of quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$, depends upon the value of the expression $b^2 - 4ac$ i.e. discriminant.

The two roots obtained using the Discriminent method, are as follows:

$$\alpha = \frac{-b + \sqrt{D}}{2a}, \qquad \beta = \frac{-b - \sqrt{D}}{2a}$$

The nature of the roots and their value has been summarised in the table below:

S. No.	Find D	Nature of Roots	Value of the roots
1.	D > 0	Real and distinct	$\frac{-b \pm \sqrt{D}}{2a}$
2.	D = 0	Real and equal (or repeated roots)	Each $= \frac{-b}{2a}$
3.	D < 0	No real roots	None
4.	$b = 0, \frac{c}{a} < 0$	Equal in magnitude but opposite in sign real and distinct.	$\sqrt{\frac{-c}{a}}, -\sqrt{\frac{-c}{a}}$
5.	c = 0	Real and distinct one being zero	0, -b/a
6.	c = a, $b^2 – 4ac > 0$	Real distinct and reciprocal to each other	-

Important Results:

Let a and b are two real numbers then

(1) If $a \geq b \Leftrightarrow -a \leq -b$ for eg, $2 \leq 3$ but $-2 \geq -3$

(2) If $ax + b \leq 0 \Rightarrow x \leq -\frac{b}{a}$

(3) If $ax + b \geq 0 \Rightarrow x \geq \frac{-b}{a}$

(4) If $x^2 - a^2 \geq 0 \Rightarrow (x+a)(x-a) \geq 0$

Either $x + a \geq 0 \,\&\, x - a \geq 0$

$\Rightarrow x \geq -a \,\&\, x \geq a$

$\Rightarrow x \geq a$ (because if $x \geq a$ then it is understood that $x \geq -a$)

or $x - a \leq 0 \,\&\, x + a \leq 0$

$\Rightarrow x \leq a \,\&\, x \leq -a$

$\Rightarrow x \leq -a$ (because if $x \leq -a$ it is understood that $x \leq a$

For eq. $x \geq 5 \Rightarrow x \geq -5$

Hence $x^2 - a^2 \geq 0 \Rightarrow x \geq a \text{ or } x \leq -a$

(5) If $x^2 - a^2 \leq 0 \Rightarrow (x-a)(x+a) \leq 0$

Either $x + a \geq 0 \,\&\, x - a \leq 0$

or $x + a \leq 0 \,\&\, x - a \geq 0$

$\Rightarrow x \geq -a$ & $x \leq a$

$\Rightarrow -a \leq x \leq a$

Hence $x^2 - a^2 \leq 0 \Leftrightarrow -a \leq x \leq a$

$\Rightarrow x \leq -a$ & $x \geq a$

There are no such values of x which satisfies the above relations

Illustration 3

Determine nature of the roots of equations

Solution

$3x^2 + 11x + 10 = 0$

given a = 3, b = 11, c = 10

D = $b^2 - 4ac \Rightarrow (11)^2 - 4 \times 3 \times 10 = 121 - 120 = 1$

Since D > 0

$\Rightarrow$ Roots are real and unequal.

Illustration 4

Find the value of k for the equation $3x^2 + kx + 3 = 0$ ***have real and equal roots***

Solution

Here, a = 3, b = k, c = 3

for roots are real and equal, $b^2 - 4ac = 0$

$(k)^2 - 4 \times 3 \times 3 = 0 \Rightarrow k^2 = 36, \ k = \pm 6$

Illustration 5

Determine the value of k, (k > 0) such that both the equations $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will have real roots

Solution

$x^2 + kx + 64 = 0$

$b^2 - 4ac \geq 0$

$k^2 - 256 \geq 0$

$\Rightarrow (k+16)(k-16) \geq 0$

$\Rightarrow k \geq 16 \ or \ k \geq -16$

$x^2 - 8x + k = 0$

$b^2 - 4ac \geq 0$

$(-8)^2 - 4 \times 1 \times k \geq 0$

$64 - 4k \geq 0$

$4k \leq 64 \quad k \leq 16$

since k > 0, therefore $0 < k \leq 16$ for both equations to is have real roots both will satisfy k = 16

*8.5 RELATION BETWEEN ROOTS AND COEFFICIENT OF $ax^2 + bx + c = 0$

If α, & β are roots of $ax^2 + bx + c = 0$

than $\alpha + \beta$ (sum of roots) = $-b/a$

$\alpha\beta$ (product of roots) = c/a

(i) **Symmetric functions of roots of a quadratic equation:** Let α and β be the roots of a quadratic equation. An expression in α and β are interchangeed, is known as a symmetric function in α and β.

To evoluate a symmetric functionof the roots of a quadratic equationin terms of its coefficients, we always expres it in terms of $\alpha + \beta$ and α and β.

The following results are very useful for the same.

(i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$.

(ii) $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

(iii) $\alpha^2 - \beta^2 = (\alpha - \beta)(\alpha + \beta) = (\alpha + \beta)\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

(iv) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$

(v) $\alpha^3 - \beta^3 = (\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2)$

(vi) $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$

(vii) $\alpha^4 - \beta^4 = (\alpha + \beta)(\alpha - \beta)(\alpha^2 + \beta^2) = (\alpha + \beta)(\alpha - \beta)[\alpha + \beta)^2 - 2\alpha\beta]$

(ii) If a , b c are the sides of a triangle, then

$3(ab + bc + ca) \leq (a + b + c)^2 \leq 4(bc + ca + ab)$

(iii) If α, β are the roots of $ax^2 + bx + c = 0$ then

(a) $\alpha^2 + \beta^2 = \dfrac{b^2 - 2ac}{a^2}$

(b) $\alpha - \beta = \dfrac{\sqrt{b^2 - 4ac}}{a}$

(c) $\alpha^2 - \beta^2 = \dfrac{-b}{a^2}\sqrt{b^2 - 4ac}$

(d) $\alpha^3 - \beta^3 = \dfrac{3abc - b^3}{a^3}$

(e) $\alpha^4 - \beta^4 = \dfrac{b^4 - 4ab^2c + 2a^2c^2}{a^4}$

(f) $\alpha^6 + \beta^6 = \dfrac{9a^2b^2c^2 + b^6 - 6ab^4c + 2a^3c^3}{a^6}$

(g) $\dfrac{\alpha + \beta}{\alpha^{-1} + \beta^{-1}} = \dfrac{c}{a}$

(h) $\sqrt{\dfrac{\alpha}{\beta}} + \sqrt{\dfrac{\beta}{\alpha}} = \dfrac{-b}{\sqrt{ac}}$

(iv) If one root of $ax^2 + bx + c = 0$ is twice the other then $2b^2 = 9ac$.

(v) If one root of the equation $x^2 + px + q = 0$ is three times the other then $16q = 3p^2$.

(vi) If the roots of the equation $x^2 - px + q = 0$ are consecutive numbers $p^2 = 4q + 1$

(vii) (a) If the ratio of the roots of the equation $ax^2 + bx + c = 0$ is m : n then $(m + n)^2ac = mnb^2$

(b) If one root of the quadratic equation is k times the other then $(k + 1)^2ac = kb^2$.

(viii) If the roots of the equation $x^2 - px + q = 0$ differ by unity then $p^2 = 4q + 1$

(ix) If unity is a root of $ax^2 + bx + c = 0$ then the other root is $\dfrac{c}{a}$.

(x) Roots are equal in magnitude but opposite in sign if $b = 0$

(xi) Both roots are zero if b = c = 0.

(xii) Imaginary Root = *i* (IOTA) (Greek word)

$\sqrt{(-1)} = i$ $i^2 = -1$ $i^3 = -i$ $i^4 = 1$

*8.6 EQUATION REDUCIBLE TO QUDRATIC FORM

Illustration 6

Solve the equation for x

$$\left(\frac{x}{x+1}\right)^2 + 6 - 5\left(\frac{x}{x+1}\right) = 0;\ x \neq 0$$

Ans. –3/2, –2

[Hint: Let $\frac{x}{x+1} = y$]

Illustration 7

Solve the equation for x: $3\sqrt{\frac{x}{5}} + 3\sqrt{\frac{5}{x}} = 10$

Ans. $\left\{\frac{5}{9}, 45\right\}$

[Hint: Let $\sqrt{\frac{x}{5}} = y$ **.]**

Illustration 8

Solve the equation for x: $\left(x^2 + \frac{1}{x^2}\right) - 3\left(x - \frac{1}{x}\right) - 2 = 0 \quad x \neq 0$

Ans. $\left\{\pm 1, \frac{3 \pm \sqrt{13}}{2}\right\}$

[Hint: Put $x - \frac{1}{x} = y$ **]**

llustration 9

Solve the equation for x: $x^4 + 2x^3 - 13x^2 + 2x + 1 = 0$

Ans. $\left\{\frac{-5 \pm \sqrt{21}}{2}\right\}, \frac{3 \pm \sqrt{5}}{2}$

[Hint: Divide both sides by x^2]

*8.7 TO FORM A QUADRATIC EQUATION WITH ROOTS α AND β

A Quadratic equation with roots α & β is given by.

x^2 – (sum of roots) x + product of roots = 0

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

*8.8 CONDITION FOR COMMON ROOTS

Given equations $ax^2 + bx + c = 0$ and $a_1x^2 + b_1x + c_1 = 0$

(i) To have one common root

$(bc_1 - b_1c)(ab_1 - a_1b) = (a_1c - ac_1)^2$

(ii) condition for the equations have reciprocal roots

$ax^2 + bx + c = 0$ and $a_1x^2 + b_1x + c_1 = 0$

$\frac{b}{b_1} = \frac{c}{a_1} = \frac{a}{c_1}$ is required condition.

(iii) condition for the equations have both common roots

$ax^2 + bx + c = 0$ and $a_1x^2 + b_1x + c_1 = 0$ to have both roots common

$\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1}$

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EXERCISE-I

- **Solve using Factorisation Method (for Q. 1 to 2):**

1. $3a^2x^2 + 8abx + 4b^2 = 0 \; a \neq 0$
2. $ax^2 + (4a^2 - 3b)x - 12ab = 0$
3. Solve the following equation by the method of completion of square: $2x^2 - 2\sqrt{6}x + 3 = 0$
4. Solve the following equations by using formula:

   (i) $x^2 - 8x + 15 = 0$ (ii) $9x^2 - 3(a^2 + b^2)x + a^2b^2 = 0$
5. If (–4) is root of $x^2 + px - 4 = 0$ and $x^2 + px + q = 0$ has equal roots, find p and q.
6. Find k so that $4x^2 - 2x + k = 0$ has one root as reciprocal of other.
7. If α, β are roots of $2x^2 - 3x + 1 = 0$, find value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$.
8. If α, β are the roots of the equation $ax^2 + bx + c = 0$, find

   (i) $1/\alpha + 1/\beta$ (ii) $\alpha^2 - \beta^2$ (iii) $\alpha^2\beta + \alpha\beta^2$

   (iv) $\alpha^3 - \beta^3$ (v) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ (vi) $\frac{\alpha^3}{\beta} + \frac{\beta^3}{\alpha}$
9. For what k, $x^2 - (3k - 1)x + (2k^2 + 2k - 11) = 0$ has real and equal roots?
10. Solve, using quadratic formula for the value of x, $x^2 - 4x + 1 = 0$.
11. The sum of the roots of the equation $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$ is zero. Prove that the product of the roots is $-\frac{1}{2}(a^2 + b^2)$.
12. Solve the equation: $7^{1+x} + 7^{1-x} = 50$
13. Determine k so that the equation $2x^2 - kx + 1 = 0$ have concident roots.
14. Solve the equation: $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$
15. If the roots of the equation $p(q - r)x^2 + q(r - p)x + r(p - q) = 0$ be equal, show that $1/p + 1/r = 2/q$.
16. Find the solution of the equation: $4x^4 - 4x^3 - 7x^2 - 4x + 4 = 0$.
17. The number of roots of the quadratic equation $8\sec^2\theta - 6\sec\theta + 1 = 0$
18. If roots of the equation $ax^2 + 2(a + b)x + (a + 2b + c) = 0$ are imaginary then roots of the equation $ax^2 + 2bx + c = 0$ are.
19. Find the condition that one root of $ax^2 + bx + c = 0$. Shall be ‘n’ times the other.

20. If α and β are roots of $x^2 - 2x + 3 = 0$, then the equation whose roots are $\frac{\alpha-1}{\alpha+1}$ and $\frac{\beta-1}{\beta+1}$ will be.

EXERCISE-II

(Note: Questions based on 5 marks each)

1. Solve for x : $4x^2 - 2(a^2 + b^2)x + a^2b^2 = 0$
2. Solve for x : $4x^2 - 4a^2x + (a^4 - b^4) = 0$
3. Solve for x : $\left[\frac{2x}{x-5}\right]^2 + \left[\frac{10x}{x-5}\right] - 24 = 0 \quad x \neq 5$
4. Solve for x : $2\left[\frac{2x+3}{x-3}\right] - 25\left[\frac{x-3}{2x+5}\right] = 5$ given that $x \neq 3 \,\&\, x \neq \frac{-3}{2}$
5. Solve $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}, a \neq 0, b \neq 0, x \neq 0 \,\&\, x \neq -(a+b)$
6. If one root of the quatradic equation $3x^2 - kx - 2 = 0$ is 2 find the value of k, and the other root.
7. For what value of k, does the quadratic equation $9x^2 + 8kx + 16 = 0$ has equal roots?
8. Solve the following quadratic equations $9x^2 - 9(a + b)x + \left[2a^2 + 5ab + 2b^2\right] = 0$
9. Solve the quadratic equation by using formula $9x^2 - 3(a+b)x + ab = 0$
10. If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots show that $c^2 = a^2(1+ m^2)$
11. If the roots of the equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ are equal prove that $2a = b + c$
12. Find two consecutive numbers whose squares has the sum 85
13. Two numbers differ by 2 and their product is 360, find the numbers
14. 300 Apples are distributed equally among a certain number of students. Had these been 10 more students, each would have been received one apple less. Find the number of students.
15. The length of the hypotenuse of a right angled triangle exceeds the length of the base by 2 cm and exceeds twice the length of the altitude by 1 cm. Find the length of each side of the triangle.
16. The hypotenuse of a grassy land in the shape of a right triangle in 1 metre more than twice the shortest side. If the third side is 7 meters more than the shortest side, find the sides of grassy land.
17. A moterboat whose speed in still water is 15 km/hr. goes 30 km down stream and comes back to the starting point in a total time of 4 hrs. and 30 minutes find the speed of the stream.

18. A train takes 2 hours less for a journey of 300 km, If its speed is increased by 5 km/hr from its usual speed. Find the usual speed of train.
19. A plane left 30 minutes later than the scheduled time and an order to reach the destination 1500 km away in time, it had to increase the speed by 250 km/hr from the usual speed. Find the usual speed.
20. A two digit number is such that the product of its digits is 35. when 18 is added to the number the digits interchange their places. find the number.

EXERCISE - III

SECTION-A

- **Multiple choice question with one correct answers**

1. Roots of the quadratic equation
$x^2 - 5x - 6 = 0$ are
(A) equal but negative (B) unequal but of same signs
(C) unequal but of opposite signs (D) equal but positive

2. The quadratic equation where one root is $3+2\sqrt{3}$ is
(A) $x^2 - 6x - 3 = 0$ (B) $x^2 + 6x - 3 = 0$
(C) $x^2 + 6x + 3 = 0$ (D) $x^2 - 6x + 3 = 0$

3. If one root of the equation $ax^2 + bx + c = 0$ is the reciprocal of other then
(A) a = b (B) b = c (C) a = c (D) a = –c

4. Given that $f(x) = 3x^4 - 5x^3 + 8x^2 - 6x + 8$ and $g(x) = x^2 - 2x + 2$. Then how many real roots does the equation $\dfrac{f(x)}{g(x)} = 0$ have?
(A) 1 (B) 2 (3) 3 (D) 4

5. The number of real roots of equation
$$(a^2+b^2)x^2+2a(\sqrt{b^2+c^2})x+a^2+c^2=0$$
where, a, b and c are non zero is
(A) 0 (B) 1 (C) 2 (4) 4

6. If the roots of the equation $x^2 - 4x + 1 = 0$ are in the ratio p : q then the value of $\sqrt{\dfrac{p}{q}}+\sqrt{\dfrac{q}{p}}$ is
(A) 0 (B) 4
(C) $2\sqrt{3}$ (D) Cannot be determined

7. How are the roots of the quadratic equations $ax^2 + bx + c = 0$ and $cx^2 + bx + a = 0$ are related?
   (A) No definite relation exist between the roots.
   (B) The roots of second equation are the sum and the difference of the roots of the first equation.
   (C) The roots of the one equation are the reciprocals of the roots of the other equation
   (D) The roots of the first and the second equations are equal in magnitude and opposite in sign.

8. The condition that the equation $x^2 + px + q = 0$ whose one root is the cube of the other root is
   (A) $p = q^{1/4}[1 - q^{1/2}]$ (B) $-p = q^{1/2}[1 - q^{1/4}]$
   (C) $-p = q^{1/4}[1 + q^{1/2}]$ (D) $p = q^{1/2}[1 + q^{1/4}]$

SECTION-B

- **Match the following (one to one)**

Column-I and **column-II** contains **four** entries each. Entries of column-I are to be matched with some entries of column-II. Only One entries of column-I may have the matching with the same entries of column-II and one entry of column-II Only one matching with entries of column-I

1.

Column–I (Quadratic equation)	Column–II (Pair of roots)
(A) $2x^2 + x - 6 = 0$	(P) (2,3)
(B) $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$	(Q) $\left(-\frac{3}{2}, \frac{3}{5p}\right)$
(C) $10px^2 - 6x + 15px - 9 = 0;\ p \neq 0$	(R) $\left(-4\sqrt{3}, \frac{2}{\sqrt{3}}\right)$
(D) $x^2 - 5x + 6 = 0$	(S) $\left(\frac{3}{2}, -2\right)$

EXERCISE - IV

SECTION-A

1. If the quadratic equation $x^2 + ax + b = 0$ and $x^2 + bx + a = 0\ (a \neq b)$ have a common root, then the numerical value of a + b is
   (A) 1 (B) 2 (C) –1 (D) None

2. The sum of all the real roots of the equation $|x-2|^2 + |x-2| - 2 = 0$
   (A) 2 (B) 3 (C) 4 (D) 1

3. Both the roots of the equation
   (x– b) (x – c) + (x– a) (x – c) + (x– a) (x – b) are always
   (A) positive (B) negative (C) real (D) None

4. The number of real solutions of the equation $|x|^2 - 3|x| + 2 = 0$

(A) 4 (B) 1 (C) 3 (D) 2

5. The equation has $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$

(A) no root (B) one root
(C) two equal roots (D) Infinitely many solution

6. Let α, β be the roots of the equation $(x - a)(x - b) = c \quad c \neq 0$
Find the roots of the equation $(x - \alpha)(x - \beta) + c = 0$ are

(A) a and c (B) b and c (C) a and b (D) a+c, b+c

7. The equation has $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$

(A) no solution (B) one solution (C) two solutions (D) None

8. The roots of the euqation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3 then

(A) $a < 2$ (B) $2 \le a \le 3$ (C) $3 \le a \le 4$ (D) $a > 4$

9. One roots is square of the other root of the equation $x^2 + px + q = 0$, then the relation between p and q is

(A) $p^3 - q(3p-1) + q^2 = 0$ (B) $p^3 - q(3p+1) + q^2 = 0$
(C) $p^3 + q(3p-1) + q^2 = 0$ (D) $p^3 + q(3p+1) + q^2 = 0$

10. If p, q, r are +ve and in A.P. Show that the roots of quadratic equation $px^2 + qx + r = 0$ are all real for

(A) $\left|\frac{r}{p} - 7\right| \ge 4\sqrt{3}$ (B) $\left|\frac{p}{r} - 7\right| \ge 4\sqrt{3}$ (C) All p & r (D) no p & r

SECTION-B

- **Multiple choice question with one or more than correct answers**

1. Consider the following statements.
If the quadratic equations, $x^2 + ax + 2 = 0$ and $x^2 + x + b = 0$ have a common root $x = 1$, then

(A) $a + b = 5$ (B) $ab = 6$ (C) $\frac{a}{b} = \frac{3}{2}$ (D) $a - b = 1$

SECTION-C

- **Comprehension**

The distance of road between the towns A and B is 216 km and by rail it 208 km. A car at a speed of x km/hr and the train travels at a speed which is 16 km/hr faster than the car. Calculate.

1. The time taken by the car to reach town B from A, in terms of x

(A) $\frac{216}{x}$ hrs. (B) $\frac{216}{x^2}$ hrs (C) $\frac{206}{x^2}$ (D) None

2. The time taken by the train to reach town B from A in terms of x.

(A) $\frac{208}{16+x^2}$ hrs. (B) $\frac{208}{16+x}$ hrs (C) $\frac{206}{16+x}$ (D) None

3. If the train taken 2 hours less than the car to reach town B, obtain an equation is *x* is solve it.
(A) 16 km/hr (B) 26 km/hr (C) 36 km/hr (D) None

SECTION-D

- **Match the following (one to many)**

Column-I and **column-II** contains **four** entries each. Entries of column-I are to be matched with some entries of column-II. One or more than one entries of column-I may have the matching with the same entries of column-II and one entry of column-II may have one or more than one matching with entries of column-I

1.

Column–I *(find k so that roots of equation)*		**Column–II** *(k)*
(A) $3kx^2 - 4kx + 4$ are real & equal	(P)	(0
(B) $(k-2)x^2+2(2k-3)x+5k-6=0$ have repeated roots	(Q)	3
(C) $ax^2 + 8kx + 16 = 0$ are real & equal	(R)	–3
(D) $x^2-6x + 3k = 0$ has coincident roots	(S)	1

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Answers

EXERCISE-I

1. $\left(\frac{-2b}{a},\frac{-2b}{3a}\right)$ 2. $\left(-4a,\frac{3b}{a}\right)$ 3. $\left(\frac{\sqrt{6}}{2},\frac{\sqrt{6}}{2}\right)$ 4. (i) (5, 3) (ii) $\left(\frac{a^2}{3},\frac{b^2}{3}\right)$

5. p = 3, q = 9/4 6. k = 4 7. 5/2

8. (i) –b/c (ii) $-\frac{b\sqrt{b^2-4ac}}{a^2}$ (iii) $\frac{-bc}{a^2}$ (iv) $\frac{(\sqrt{b^2-4ac})(b^2-ac)}{a^3}$

(v) $\frac{(3abc-b^3)}{a^2c}$ (vi) $\frac{(b^4-4acb^2+2a^2c^2)}{a^3c}$ 9. k = 9, 5 10. $x=2\pm\sqrt{3}$

12. (–1, 1) 13. $k=2\sqrt{2},-2\sqrt{2}$ 14. $(2\pm 2\sqrt{3})$ 16. $\left\{\frac{1}{2},2\right\}$

17. 0 18. complex 19. $nb^2(1+n^2)ac$ 20. $3x^2-2x+1=0$

EXERCISE-II

1. $\frac{a^2}{2},\frac{b^2}{2}$ 2. $x=\frac{a^2+b^2}{2}$ or $x=\frac{a^2-b^2}{2}$ 3. x = 4, 15

4. x = 1 & 6 5. x = –a & –b 6. k =5 and other root is –1/3

7. $x=\frac{2a+b}{3}$ & $x=\frac{a+2b}{3}$ 8. $\frac{a}{3},\frac{b}{3}$ 9. k = ±3

12. 6 & 7 or – 7 & – 6 13. 18 and 20 14. 50

15. Base = 15 cm, altitude 8 cm, Hypotenuse = 17 cm 16. 8 m, 17 m and 15 m

17. speed of stream 5 km/hr. 18. 25 km/hr. 19. 750 km/hr

20. 57

EXERCISE-III

SECTION-A

1.	(C)	2.	(A)	3.	(C)	4.	(B)	5.	(A)	6.	(B)
7.	(C)	8.	(C)								

SECTION-B

1. (A)–(S), (B)–(R), (C)–(S), (D)–(P)

EXERCISE-IV

SECTION-A

1.	(C)	2.	(C)	3.	(A)	4.	(A)	5.	(A)	6.	(C)
7.	(A)	8.	(A)	9.	(A)	10.	(B)				

SECTION-B

1. (A,B,C)

SECTION-C

1.	(A)	2.	(B)	3.	(C)

SECTION-D

1. (A)-(P,Q), (B)-(P,S), (C)-(Q,R), (D)-(R)

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