9 COORDINATE GEOMETRY

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9.1 INTRODUCTION

Coordinate geometry is that branch of geometry in which two numbers, called coordinates, are used to calculate the position of a point in a plane.

In this chapter we shall define the coordinates of a point in a plane with reference to two mutually perpendicular lines in the same plane. We shall study rectangular coordinate system and also how a straight line or a curve in a plane can be represented by an algebric equation. Rene dascartes is known as the father of analytical or coordinate geometry.

9.2 CARTESIAN COORDINATES

Representation of points in the plane by ordered pairs of real numbers called cartesian coordinates of that point.

9.3 CO-ORDINATE AXES AND CARTESIAN PLANE

XoX' and YoY' be two mutually perpendicular line through a point O in the plane of a graph paper.

(1) Both lines are perpendicular to each other

(2) Both lines intersect at their zeroes (denoted by O).

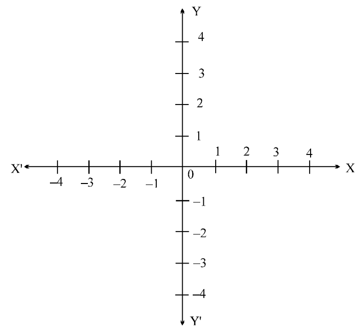
These two mutually perpendicular number lines are called co-ordinate axes or rectangular axes.

The plane in which the coordinate axes are drawn is called co-ordinate plane or cartesian plane.

(i) x axis: The horizontal line XoX′ is called x axis or Abscissa

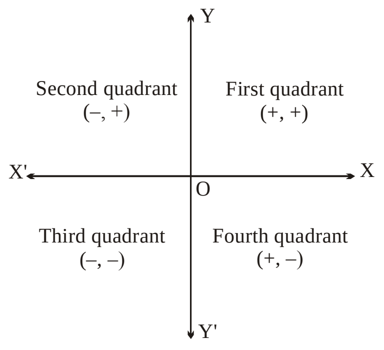
(ii) y-axis: The vertical line YoY′ is called y-axis or Ordinate.

(iii) origin : The point of intersection O of two axes is called origin.



9.4 QUADRANT

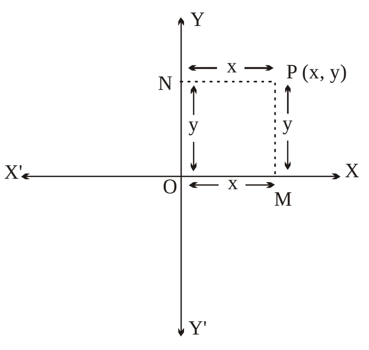
The co-ordinate axes XoX′ and YoY′ divide the co-ordinate plane into four regions, which are called quadrants. The regions XoY, YoX′ X′oY′ and Y′o X are respectively known as the first second, third and fourth quadrants.



(i) Ist Quadrant: $X > 0, Y > 0$
(ii) IInd Quadrant: $X < 0, Y > 0$
(iii) IIIrd Quadrant: $X < 0, Y < 0$
(iv) IVth Quadrant: $X > 0, Y < 0$

9.5 CARTESIAN CO-ORDINATES OF A POINT

Let p be any point in the cartesian plane.
(a) Through the point p, draw PM perpendicular to XoX′ and PN perpendicular to YoY′.



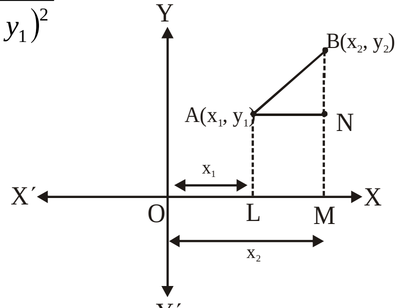
(b) We have OM = PN
= the distance of the point p from the y-axis
= x (say)
= X co-ordinate or abscissa of the point P.
We have ON = PM
= the distance of the point p from the x-axis
= y (say)
= y-co-ordinate or ordinate of the point P.
(c) Thus,the coordinate of the point P= (abscissa, ordinate) = (x, y)

Remarks:
(i) (x, y) is an ordered pair in which x and y cannot be interchanged $(a, b) \neq (b, a)$
(ii) If we take a point on x-axis then distance of this point from x-axis is 0, therefore, ordinate of this point is 0, i.e. at x-axis y = 0.
(iii) We take a point on y-axis then its distance from y-axis is 0. and therefore the x-coordinate or Abscissa of this point is 0.i.e. at y-axis x = 0.

9.5.1 Distance Formula

The distance between two points A(x_1, y_1) and B(x_2, y_2) is given by the formula

AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



Remarks:

(i) The distance of a point P(x, y) from the origin O (0, 0) is **OP** $= \sqrt{x^2 + y^2}$

SOME IMPORTANT POINTS :

(A) Three points will form

(i) A right angle triangle if sum of squares of any two sides is equal to square of third largest side.

(ii) An equilateral triangle, if all the three sides are equal.

(iii) An isosceles triangle, if any two sides are equal

(iv) Be collinear or form a line, if sum of two sides is equal to third sides.

(B) When four points are given

(i) They form a square if all the sides are equal and diagonals are also equal.

(ii) They form a rectangle if the opposite sides are equal and diagonals are also equal

(iii) They form a rhombus if all the sides are equal but diagonals are not equal

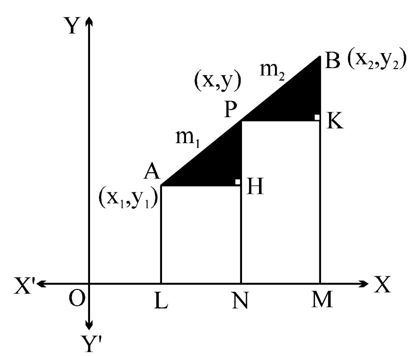
(iv) They form a parallelogram if the opposite sides are equal but diagonals are not equal

9.5.2 Section Formula

A(x_1, y_1) and B(x_2, y_2) are two points and P(x, y) divides AB internally in the ratio $m_1 : m_2$ then

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \qquad y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

Hence the coordinate of P are $\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$



Remarks

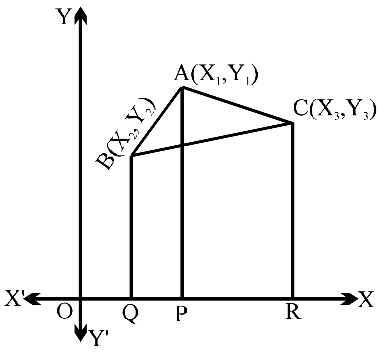
(i) If p is the mid point of AB then it divides AB in the ratio 1 : 1 so

its coordinate are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

9.5.3 Area of Triangle

The area of a triangle, the coordinates of whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is

$$= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$



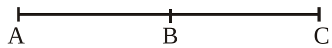
9.5.4 Conditions of collinearity of three points

Three points $A(x_1, y_1)$ $B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear iff

(i) Area of a triangle ABC is zero

(ii) Distance between A and B + distance between B and C = distance between A and C

(AB + BC = AC)



(iii) Slope of AB = slope of BC = Slope of AC

9.5.5 Centroid of a Triangle

When a vertex of a triangle is joined to the mid point of the opposite side, we get a median. The point of intersection of the median is called the centroid of the tiangle. The centroid divides any median in the ratio 2 : 1.

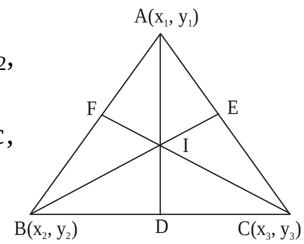
The coordinates of the centroid of a triangle whose vertices are $A(x_1, y_1)$,$B(x_2, y_2)$ and $C(x_3, y_3)$ is given by:

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

9.5.6 Incentre of a Triangle

The point of intersection of the internal bisectors of the angles of a triangle is called the incentre.

The coordinates of the incentre of a triangle whose vertics are $A(x_1, y_1)$,$B(x_2, y_2)$ and $C(x_3, y_3)$ are $\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$ where AB = c, AC = b , BC = a.

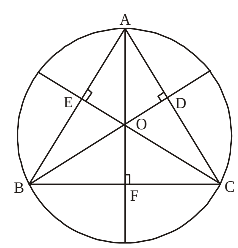


9.5.7 Circumcentre

It is point of intersection of perpendicular bisector of the sides of a triangle circle it is also the centre of a passing vertices of the triangle if O is the circumcentre of any triangle ABC then

$OA^2 = OB^2 = OC^2$

Remarks : If a triangle is a right angle then its circumcentre is the mid point of hypotenuse.



9.5.8 Orthocentre

It is the point of intersection of perpendicular drawn from vertices on opposite side (called altitudes) of a triangle is called orthocentre.

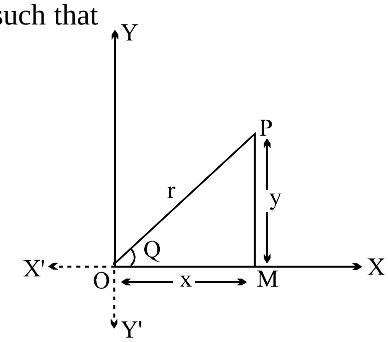
(i) If a triangle is right angled triangle then orthocentre is the point where right angle is formed.

(ii) If the triangle is equilateral, the centroid, incentre, orthocentre and circum-centre coincides.

(iii) In an isosceles triangle centroid, orthocentre, in centre, circum-centre lies on the same line.

*9.6 POLAR COORDINATES

Let OX be any fixed line which is usually called the initial line and O be a fixed point on it. Let P be any point whose cartesian coordinates referred to rectangular axes are x and y and whose polar coordinates, referred to O as pole are p(r, Q) Draw PM perpendicular to OX such that



OM = x PM = y

$\angle MOP = \theta$ and OP = r

we have

$x = r\cos\theta$ (i)

$y = r\sin\theta$ (ii)

where $r = \sqrt{x^2 + y^2}$ (iii)

(ii) divided by (i) then we get

$\tan\theta = y/x$

Remark:

(i) $x = r\cos\theta$ $y = r\sin\theta$ express the cartesian coordinates in term of the polar coordinate

(ii) $r = \sqrt{x^2 + y^2}$ and $\tan\theta = y/x$ express the polar in terms of the cartesian coordinates

*9.7 IMAGE OF AN OBJECT IN A MIRROR

When an object is placed in front of a plane mirror then its image is formed at the same distance behind the mirror as the distance of the object from the mirror.

9.7.1 Reflection

The transformation R_1 which maps a point 'P' to its image P in a given line (or point) Q, is called a reflection in *l*

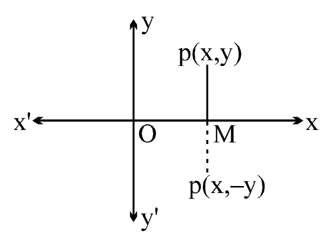
We shall represent

(i) Reflection in X-axis by R_X

(ii) Reflection in Y-axis by R_Y

(iii) Reflection in the origin by R_O

(i) Reflection in x-axis:



Let P(x, y) be a point in a plane. Draw PM $\perp$ OX meeting at M. Produce PM to P^1 such that $MP = MP^1$ then P^1 is the Image of P when reflected in X-αxis. Clearly the co-ordinates of $P^1(x, -y)$

$\therefore$ P(x, y) when reflected in X-αxis have the image $p^1(x, -y)$

$\therefore R_X(x, y) = (x, -y)$

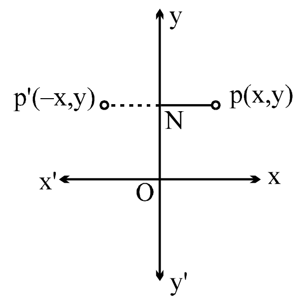
(ii) Reflection in y-axis

Let P (x,y) be a point in a plane. Draw PN $\perp$ OY, meeting it at N product PN to P′such that NP = NP^1. Then, P^1 is the image of P when reflected in Y-αxis. Clearly the Co-ordinates of P^1 are

P^1 (–x, y)

∴ P(x,y) when reflected in X-axis, have the image P^1 (–x,y)

∴ $R_Y(x,y) = (-x,y)$



(iii) Reflection in the origin

Let P(x, y) be a point in a plane. Join PO and produce it to P^1 such that OP^1 = OP. Then P^1 is the images of P when reflected In the origin. Clearly the Co-ordinate of P^1 are P^1 (–x, –y)

∴ P(x, y) when reflect in the origin has the Image P^1 (–x, –y)

∴ Ro (x,y) = (–x, –y)

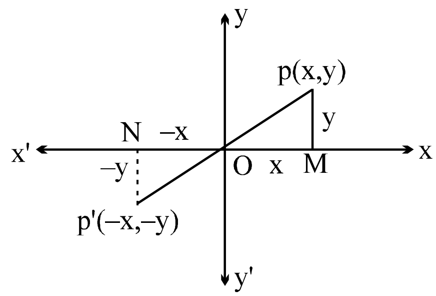


Illustration 1

Point P (a, b) is reflected in the x-axis to P^1*(5, 2) write down the values of a and b*

Solution

We know that

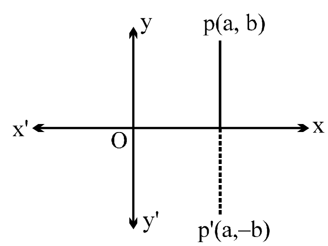
R_λ (a, b) = (a, –b)

∴ R_λ (a, b) = (5, –2)

⇒ (a, b) = (5, –2)

a = 5, b = 2

Hence the Co-ordinate of P (5, 2)



Some Important Results and Properties

- A triangle is isosceles if any two of its sides or medians are equal.
- The lines joining middle points of opposite sides of a quadrilateral bisect each other.
- In a traingle, ABC, If AD is the median drawn to the side BC, then $\mathbf{AB^2 + AC^2 = 2 (AD^2 + CD^2)}$
- If G is the centroid of a triangle ABC then $AB^2 + BC^2 + CA^2 = 3(GA^2 + GB^2 + GC^2)$.
- If G is the centroid of a triangle ABC then the area of each of the triangle AGB, BGC and CGA are equal and each is equal to one third of the area of triangle ABC.
- The incentre of a triangle is equidistant from the vertices of the triangle.
- **For an equilateral triangle orthocenter, circum centre, centroid and incentre are the same.**
- The area of the triangle joining the mid - points of the sides of the triangle ABC is $\frac{1}{4}$ area of triangle ABC.
- The fourth vertex of a parallelogram where the three consecutive vertices are (x_1, y_1) (x_2, y_2), (x_3, x_3) is $(x_1 + x_3 - x_2, y_1, + y_3 - y_2)$

EXERCISE-I

1. The vertices of a triangle ABC are A(4, 6) , B(1,5) and C(7, 2). A line is drawn to intersect sides AB and AC at D and E respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$ Calculate area of Δ ADE and compare it with area of ΔABC.

2. If A,B,C are three points with coordinates A $(at^2, 2at)$ $B\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$ and C(a, o) respectively show that $\frac{1}{AC} + \frac{1}{BC} = \frac{1}{a}$ where a > 0

3. The vertices of a triangle ABC are A(–2, –5) B(–2, 11) and C(10, –1), D and E are mid- points of AB and AC respectively. show that $DE = \frac{1}{2} BC$

4. If points A (1, 2) B(1, 3) and C(x, y) are such that AB = BC and A, B, C lie in the same straight line. Then find the values of x and y

5. If x, y, z are the mid-points of sides PQ, QR and RP of a triangle PQR whose vertices are P(0, 0) Q(6, 0) and R(0, 8) find

   (i) Perimeter of ΔXYZ (ii) Perimeter of ΔPQR

6. A(3, 2) B(–2, 1) are two of the vertices of a triangle ABC whose centroid is G$\left(5/3, -1/3\right)$. Find the coordinate of the third vertex C of the triangle.

7. Find the In-centre of the triangle ABC where A(0, 0), B(4, 0) and C(0, 3)

8. If (α, β) be the middle point of the line segment joining the points (3, 4) and (5, –9) then show that $\alpha + 2\beta + 1 = 0$

9. Determine the ratio in which the line 3x + y – 9 = 0 divides the segment joining the points (1, 3) and (2, 7)

10. In what ratio is the line segment joining the points A(6,3) and B(–2, –5) divided by the x-axis.

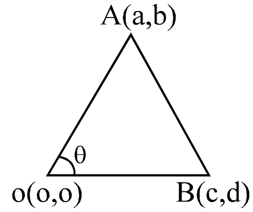
EXERCISE-II

1. Prove that the distance of the point P (acosα, asinα) from the origin is independent of α

2. If G be the centroid of a triangle ABC prove that $AB^2 + BC^2 + CA^2 = 3(GA^2 + GB^2 + GC^2)$

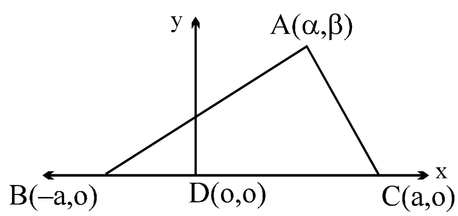
3. If the line segment joining the point A (a, b) and B (c, d) subtends an angle θ at the origin prove that

$$\cos\theta = \frac{ac + cd}{\sqrt{(a^2 + b^2)(c^2 + d^2)}}$$



4. If P and Q are two points whose coordinate are $(at^2, 2at)$ and $\left(\frac{a}{t^2}, \frac{2a}{t}\right)$ respectively and S is the point (a, o) show that $\frac{1}{SP} + \frac{1}{SQ}$ is indipendent of t.

5. If D is the middle point of ABC in any ΔABC, then prove that $AB^2 + AC^2 = 2(AD^2 + BD^2)$



6. If the distance of the point P(x, y) from A(a, o) be a + x prove that $y^2 = 4ax$
7. Find the centre of the circle of the circle passing through the points A(3, –7) B(6, 6) and C(3, 3)
8. If points (10, 5), (8, 4) and (6, 6) are the mid-points of the sides of a triangle, find its vertices.
9. If the centroid of the triangle formed by the points P(a, b), Q(b, c) and R(c, a) is at the origin. What is the value of

   (i) a + b + c (ii) $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$

10. Find the coordinate of the points which divides internally, the line joining the point (a + b, a – b) to the point (a – b, a + b) in the ratio a : b

EXERCISE-III

SECTION-A

- **Multiple choice question with one correct answers**

1. The perimeter of the triangle whose vertices are (-1, 4) (-4, -2), (3, -4) will be:
   (A) 38 (B) 16 (C) 42 (D) None of these
2. The incentre of the triangle with vertices $(1, \sqrt{3})$ (0, 0) and (2, 0) is

(A) $\left(1, \sqrt{3}/2\right)$ (B) $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$ (C) $\left(\frac{2}{3}, \sqrt{3}/2\right)$ (D) $\left(1, \frac{1}{\sqrt{3}}\right)$

3. If P(1, 2) Q(4, 6) R(5, 7) and S(a, b) are the vertices of a parallelogram PQRS than
(A) a =2 b = 4 (B) a =3 b = 4 (C) a = 2 b = 3 (D) a = 3 b = 5

4. If the points (a, 0) (0, b) and (1, 1) are collinear then,
(A) $\frac{1}{a^2}+\frac{1}{b^2}=1$ (B) $\frac{1}{a^2}-\frac{1}{b^2}=1$ (C) $\frac{1}{a}+\frac{1}{b}=1$ (D) $\frac{1}{a}-\frac{1}{b}=1$

5. If (3, –4) and (–6, 5) are the extremities of the diagonal of a parallelogram and (–2, 1) is its third vertex, then its fourth vertex is
(A) (–1, 0) (B) (0, –1) (C) (–1, 1) (D) None of these
(Hint: the diagonal of a parallelogram biscet each other)

6. The area of the triangle with vertices at the point (a, b + c) (b, c + a), (c, a + b) is
(A) 0 (B) a + b + c (C) ab + bc + ca (D) None of these

7. A point A lies on x-axis and abscissa p + q. Another point B lies on y-axis and has ordinate p – q. Find the distance AB between them.
(A) $2\sqrt{(p^2+q^2)}$ units (B) $\sqrt{2(p^2+q^2)}$ units (C) $2\sqrt{(p^2-q^2)}$ (D) None of these

8. The centroid, circumcentre, orthocentre in a triangle are:
(A) always coincident (B) always collinear
(C) always form a triangle
(D) coincident in a equilateral triangle otherwise collinear

9. A triangle ABC, right angled at A has points A and B as (2, 3) and (0, –1) respectively. If BC = 5 units then the point C is
(A) (–4, 2) (B) (4, 2) (C) (3, –3) (D) (0,–4)

10. Area of quadrilateral formed by the lines | x | + | y | = 1 is:
(A) 4 (B) 2 (C) 8 (D) None of these

SECTION-B

- **Match the following (one to one)**

Column-I and **column-II** contains **four** entries each. Entries of column-I are to be matched with some entries of column-II. Only One entries of column-I may have the matching with the some entries of column-II and one entry of column-II Only one matching with entries of column-I

1. Column II give the area of triangles whose vertices are given in column I match them correctly.

Column I	**Column II**
Vertices of triangle	**Area of triangle (sq. units)**
(A) (2,3), (–1,0) (2,–4)	(P) 40
(B) (–5,–1) (3,–5) (5,2)	(Q) 24
(C) (1,–1) (–4,6) (–3,–5)	(R) 32
(D) (0,0) (8,0) (0,10)	(S) 10.5

EXERCISE-IV

SECTION-A

- **Multiple choice question with one correct answers**

1. How many squares are possible if two of the vertices of a quadrilateral are (1, 0) and (2, 0) ?
(A) 1 (B) 2 (C) 3 (D) 4

2. The line 3x + 4y = 12 cuts the axes at A and B if O is the origin the area of ΔOAB is:
(A) 12 sq. units (B) 9 sq. units (C) 24 sq. units (D) 6 sq. units

3. P(3, 1) Q(6, 5) and R (x, y) are three points such that the angle PRQ is a right angle and the area of ΔRQP =7. then the number of such points R is
(A) 0 (B) 1 (C) 2 (D) 4

4. The centroid of a triangle is (1, 4) and the coordinates of two of its vertices are (4, -3) and (-9, 7). The area of the triangle is:
(A) 183 sq. units (B) 91.5 sq. units (C) 124 sq. units (D) None of these

5. A (6, 3), B (–3, 5) C(4, –2) and D (x, 3x) are four points. If Δ DBC : Δ ABC = 1:2 then x is equal to
(A) $11/8$ (B) $8/11$ (C) 3 (D) None

6. What will be the polar co-ordinates of the points (4, 4)
(A) $(4\sqrt{2}, 30^\circ)$ (B) $(4\sqrt{2}, 45^\circ)$ (C) $(2\sqrt{2}, 45^\circ)$ (D) $(2\sqrt{2}, 30^\circ)$

7. The medians AD and BE of triangle with vertices A (o,b), B(0,0) and C(a,o) are perpendicular if
(A) $b = \pm\sqrt{2}a$ (B) $a = \pm\sqrt{2}b$ (C) b= –2a (D) a= –2b

8. The line 2x + 3y = 6 meets x axis at the point:
(A))(3, 0) (B) (0, 3) (C) (3, 2) (D) (2, 3)

9. Two mutually perpendicular straight lines through the origin from an isosceles triangles with the line 2x+y = 5. Then the area of the triangle is
(A) 5 (B) 3 (C) $5/2$ (D) 1

10. If the centroid of the triangle formed by the points (a,b) (b,c) and (c,a) is at the origin, then $a^3+b^3+c^3=$
(A) abc (B) o (C) a+b+c (D) 3 abc

SECTION-B

- **Multiple choice question with one or more than one correct answers**

1. The Co-ordinates of the fourth vertex of the parallelogram where three of its vertices are (–3, 4) (0,–4) and (5, 2) can be
(A) (8, –6) (B) (2,10) (C) (–8,–2) (D) None

2. A and B are two fixed points where Co-ordinate are (3,2) and (5,4) respectively. The Co-ordinate of a point P if ABP is an equilateral triangle
(A) $(4-\sqrt{3},\ 3+\sqrt{3})$ (B) $(4+\sqrt{3},\ 3-\sqrt{3})$ (C) $(3-\sqrt{3},\ 4+\sqrt{3})$ (D) $(3+\sqrt{3},\ 4-\sqrt{3})$

3. The three given points A, B, C are collinear:
   (A) area of ΔABC is 0
   (B) slope of AB = slope of BC = slope of AC
   (C) distance between A and B = distance between B and C = distance between A and C
   (D) the third point satisfied the equation of line passing through any two points
4. When four points are given
   (A) They form a square if all sides are equal and diagonals are also equal
   (B) They form a rectangle if the opposite sides are equal and diagonals are also equal
   (C) They form a rhombus if all sides are equal but diagonals are not equal
   (D) They form a parallelogram if opposite sides are equal but diagonals are not equal
5. If polar co-ordinate of any points are $\left(2, \pi/3\right)$ then its cartesian co-ordinates will be:
   (A) $\left(1, \sqrt{3}\right)$ (B) $\left(-1, \sqrt{3}\right)$ (C) $\left(-1, -\sqrt{3}\right)$ (D) $\left(1, -\sqrt{3}\right)$
6. Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are:
   (A) parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
   (B) perpendicular if $a_1a_2 + b_1b_2 = 0$
   (C) intersecting if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
   (D) coincident if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

SECTION-C

- **Comprehension**

The line 3x+y–9=0 divides the segment joining the points A(1,3) and B(2,7) at P.

1. In what ratio the line 3x+y–9 =0 divides the line segment AB.
   (A) 3 :4 (B) 3:2 (C) 4 :3 (D) None of these
2. Write the coordinate of the point P
   (A) $\left(\frac{10}{7}, \frac{-33}{7}\right)$ (B) $\left(\frac{10}{7}, \frac{33}{7}\right)$ (C) $\left(\frac{33}{7}, \frac{10}{7}\right)$ (D) None
3. Find the length of the line segment AB
   (A) 4 (B) 17 (C) $\sqrt{17}$ (D) $\sqrt{109}$

Answers

Exercise-I

1. Area of triangle = $\frac{15}{32}$ sq. units.
4. x =1, y = 4
5. (i) 12 units (ii) 24 units
6. (4, –4)
7. I = (1,1)
9. 3 : 4
10. 3 : 5

Exercise-II

8. (4, 5) (8, 7) (12, 3)
9. (i) 0 (ii) 3
10. $\left(\frac{a^2+b^2}{a+b}, \frac{a^2+2ab-b^2}{a+b}\right), \left(\frac{a^2-2ab-b^2}{a-b}, \frac{a^2+b^2}{a-b}\right)$

Exercise-III

Section-A

1.	(D)	2.	(D)	3.	(C)	4.	(C)	5.	(A)
6.	(A)	7.	(B)	8.	(D)	9.	(B)	10.	(A)

Section-B

1. (A)-(S), (B)-(R), (C)-(Q), (D)-(P)

Exercise-IV

Section-A

1.	(B)	2.	(D)	3.	(C)	4.	(B)	5.	(A)
6.	(B)	7.	(A)	8.	(A)	9.	(A)	10.	(D)

Section-B

1.	(A,B,C)	2.	(A,B)	3.	(A,B,C,D)	4.	(A,B,C, D)	5.	(A)
6.	(A,B,C,D)								

Section-C

1.	(A)	2.	(B)	3.	(C)