# 10 ARITHMETIC PROGRESSION



## 10.1 INTRODUCTION

In partical life, we must have observed many things follow a certain pattern, such as the petals of a sunflower, the holes of a honeycomb, the grains on a maize cob, the spirals on a pineapple on a pipe cone.

In our day-to-day life we see patterns of geometric figures on clothes, picture, posters. They make the learners motivated to form such new pattern.

For example,

→ ← → ← ?

Solution: →

Likewise number patterns are also faced by learners. In their study, number pattern play an important role in the field of mathematics.

Ex. (i) 2,4,6,8,10............then next number is 12

(ii) $4, \frac{1}{2}, \frac{1}{16}, \frac{1}{128}$...... next number is 1024

## 10.2 SEQUENCE

Sequence is a set of term which may be real complex and an algebric expression arrange in a define order according to certain rule.

Ex. 1,2,3............

$(x+1), (2x+2), (3x+3)$,............

**Progressions:** The sequences whose terms follow, certain patterns are called progressions

Like 1,3,5,7..........

**Arithmetic progression:** It is a sequence whose term decrease or increase by a fix/constant number. This constant number is called common difference of A.P. and it generally denoted by 'd'.

[$d = a_{n+1} - a_n$]

OR

A sequence is called an arithmetic progression. if the difference of a term and previous term is always same.

The difference is called the common difference of arithmetic progression.

***Illustration 1***

***Show that the sequence on defined by $a_n = 4n + 5$ is an A.P. Also find its common difference.***

***Solution***

We have $a_n = 4n+5$ ...(i)

Replacing n by (n+1) we get

$a_{n+1} = 4(n+1) + 5$

$= 4n + 4 + 5$

$a_{n+1} = 4n + 9$ ...(ii)

$d = a_{n+1} - a_n$ $\Rightarrow$ $d = (4n+9) - (4n+5)$

$d = 4$

**Remarks:**

(i) The common difference 'd' should be independent of n.

## 10.3 GENERAL TERM

a, a+d, a+2d, a+3d.........represents an arithmetic progression where a is the first term and d the common difference. This is called the general form of an A.P.

Let 'a' be the first term and 'd' be the common difference of an A.P. Then its

$n^{th}$ term is = a + (n - 1)d

**nth term of an A.P. from the end:**

If 'a' be the first term and 'd' be the common difference of an A.P. having 'm' terms. Then the $n^{th}$ term from the end is $(m - n + 1)^{th}$ term from the beginning.

[$n^{th}$ term from the end: $a_{m-n+1}$ = a + (m - n + 1 - 1)d = **a + (m - n) d**]

## 10.4 SUM TO n TERMS OF AN AP

The sum $S_n$ of n terms of an A.P. with first term 'a' and common difference 'd' is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{n}{2}[a + l]$$ where $l$ = a +(n–1)d, $l$ = last term

**Some Important results**

(i) The sum of first *n* positive integers

$S_n = 1+2+3+4+...............+n$

$$\left[S_n = \frac{n(n+1)}{2}\right]$$

(ii) The sum of square of first n positive integers

$S_n = 1^2 + 2^2 + 3^2+........+n^2$

$$\left[S_n = \frac{n(n+1)(2n+1)}{6}\right]$$

(iii) The sum of the cubes of first ‘n’ no positive integers

$S_n = 1^3 + 2^3 + 3^3 +...........+n^3$

$$\left[S_n = \left\{\frac{n(n+1)}{6}\right\}^2\right]$$

(iv) $[a_n = S_n - S_{n-1}]$

where $a_n$ = nth term, $S_n$ = sum of n terms

## *10.5 SELECTION OF TERMS AN A.P.

| No. of terms | Terms | Common difference |
|---|---|---|
| 3 | a-d, a, a+d | d |
| 4 | a-3d, a-d, a+d, a+3d | 2d |
| 5 | a-2d, a-d, a, a+d, a+2d | d |
| 6 | a-5d, a-3d, a-d, a+d, a+3d, a+5d | 2d |

***Illustration 2***

***The sum of three numbers in A.P. is 27, and their product is 504, find them***

***Answer***

The numbers are 6,9,12 and 12, 9, 6

## *10.6 ARITHMETIC MEANS

Three quantities are in arithmetical progression the middle one is said to be the arithmetic mean of the other two.

Thus a is the arithmetic mean between a–d and a + d

**(i) Insertion of a single arithmetic mean between a and b**

Let A be the arithmetic mean of a and b. Then

a, A, b are in A.P.

A–a = b–A

2A = a + b $\Rightarrow$ $A = \frac{a+b}{2}$

**(ii) Insertion of n arithmetic means between a and b**

Let $A_1, A_2, A_3 .........A_n$ be n arithmetic means between two quantities a and b.

Then

$a, A_1, A_2 .........A_n, b$ is an A.P.

$A_n = \left(a + \frac{n(b-a)}{n+1}\right)$

These are required arithmetic means between a and b.

***Illustration 3***

***Insert three arithmetic means between 3 and 19.***

***Solution***

***Note:*** Sum of 'n' arithmetic mean inserted between two numbers a and b is

$S = \frac{n}{2}(a+b)$ where n = number of arithmetic mean

## 10.7 PROPERTIES OF ARITHMETICAL PROGRESSIONS

**Property–1:** If a constant is added to or subtracted from each term of an AP, then the resulting sequence is also an AP with the same common difference.

**Property–2:** If each term of a given AP is multiplied or divided by a non-zero constant k, then the resulting sequence is also an AP with common difference kd or d/k. Where d is the common difference of the given AP.

**Property–3:** In a finite AP the sum of the terms equidistant from the beginning and end is always same and is equal to the sum of first and last term i.e. $a_k + a_{n-(k-1)} = a_1 + a_n$

For all k = 1,2,3........(n–1)

**Property–4:** Three numbers a,b,c are in AP Iff 2b = a + c

**Property–5:** A sequence is an AP Iff its nth term is a linear expression in n i.e. $a_n = An + B$ where A, B are constants. In such a case the coefficient of n in $a_n$ is the common difference of the AP.

**Property–6:** A sequence is an AP Iff the sum of its first n terms is of the form $An^2 + Bn$ where A, B are constants independent of n. In such a cases the common difference is 2A. i.e. 2 times the coefficient of $n^2$.

**Property–7:** If the terms of an AP are chosen at regular intervals then they form an AP.

***Note:***

(i) Sum of 'n' natural numbers of 5050.

(ii) If each term of an AP is increased, decreased multiplied or divided by the same non zero number then the resulting sequence is also an AP.

# *Solved Examples*

***Example 1***

***If a, b, c are in A.P., Prove that the***

***b + c, c + a, a + b are in A.P.***

***Solution***

b + c, c + a, a + b will be in A.P.

(c + a) - (b + c) = (a + b) - (c + a)

a - b = b - c

2b = a + c

Thus a, b, c are in A.P.

$\Rightarrow$ b + c, c + a, a + b will be in A.P.

***Example 2***

***If $a^2$, $b^2$, $c^2$ are in A.P. then prove that the*** $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ ***are in A.P.***

***Solution***

$$\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$$

$$\frac{a}{b+c}+1, \frac{b}{c+a}+1, \frac{c}{a+b}+1 \text{ are in A.P.}$$

$$\frac{a+b+c}{b+c}, \frac{b+c+a}{c+a}, \frac{c+a+b}{a+b} \text{ are in A.P.}$$

$$= \frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b} \text{ are in A.P.}$$

$$= \frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

$$\frac{(b-a)}{(c+a)(b+c)} = \frac{(c-b)}{(a+b)(c+a)}$$

$$= \frac{b-a}{b+c} = \frac{c-b}{a+b}$$

$b^2 - a^2 = c^2 - b^2 \quad \Rightarrow \quad 2b^2 = a^2 + c^2$

Thus $a^2$, $b^2$, $c^2$ are in A.P. $\Rightarrow \frac{a}{b+c}, \frac{b}{a+c}, \frac{c}{a+b}$ are in A.P.

***Example 3***

***If the sum of m terms of an AP is the same as the sum of its n terms, show that the sum of its (m+n) terms is zero***

***Solution***

Let 'a' be the first term and 'd' be the common difference of the given AP then

$S_m = S_n$

$\frac{m}{2}[2a+(m-1)d] = \frac{n}{2}[2a+(n-1)d]$

m[2a+(m–1)d] = n[2a+(n–1)d]

2am + m(m–1)d = 2an + n(n–1)d

2a(m–n) + {m(m–1) –n(n–1)}d = 0

$2a(m-n)+\{(m^2-n^2) -(m-n)\}d = 0$

(m–n) [2a+(m+n–1)d]=0

2a + (m+n–1)d = 0 ..(i) [$\therefore$ m+n $\neq$ 0]

Now, $S_{m+n} = \frac{m+n}{2}[2a+(m+n-1)d] = \frac{m+n}{2} \times 0 = 0$

***Example 4***

***The interior angles of a polygon are in AP. The smallest angle is 120° and the common difference is 5°. Find the number of sides of polygon.***

***Solution***

Let there be n sides of the polygon. Then the sum of its interior angles is given by

$S_n$ = (2n–4) right angles

(n–2) × 180° ..(i)

Hence the interior angles term an AP with first term a = 120° and common difference d = 5°

$S_n = \frac{n}{2}[2a+(n-1)d]$

$(n-2)\times 180 = \frac{n}{2}[2\times 120° + (n-1)\times 5°]$

$(n-2)\times 360 = n[5n+235]$ $\Rightarrow$ $n^2-2n+144=0$

$\Rightarrow$ (n–16) (n–9) = 0 $\Rightarrow$ n = 16 or n = 9

But when n = 16 the last angle $a_n = a + (n-1)d$

120 + (16–1)5°

120 + 75 = 195° which is not possible

Hence

n = 9

***Example 5***

***If the $p^{th}$ term of an AP is 1/q and $q^{th}$ term 1/p. Prove that the sum of the first pq terms is*** $\frac{1}{2}(pq+1)$

***Solution***

$p^{th}$ term of an AP = $a_p$ = a+(p–1)d

$$= \frac{1}{q} = a+(p-1)d \quad ...(i)$$

$q^{th}$ term of an AP = $a_q$ = a+(q–1)d ...(ii)

Solving equation (i) and (ii), we get

$$d[(p-1)-(q-1)] = \frac{1}{q} - \frac{1}{p}$$

$$d[p-q] = \frac{p-q}{pq} \quad \Rightarrow \left[d = \frac{1}{pq}\right]$$

Putting the value of 'd' in equation (i), we get

$$a+(p-1)\times\frac{1}{pq} = \frac{1}{q}$$

$$a+\frac{1}{q}-\frac{1}{pq} = \frac{1}{q}$$

$$\left[a = \frac{1}{pq}\right]$$

Now the sum of first pq terms we have

$$a = \frac{1}{pq} \quad d = \frac{1}{pq} \quad n = pq$$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$S_{pq} = \frac{pq}{2}\left[\frac{2}{pq}+(p_q-1)\times\frac{1}{pq}\right]$$

$$= \frac{pq}{2}\left[\frac{2}{pq}+1-\frac{1}{pq}\right] \Rightarrow \frac{pq}{2}\left[\frac{pq+1}{pq}\right]$$

$$\left[s_{pq} = \frac{1}{2}(pq+1)\right]$$

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# EXERCISE-I

1. The Ist and the last term of an AP are –4 and 146. The sum of the terms is 7171. The number of term in the AP and the common difference
2. Show the sequence <$a_n$> defined by $a_n = 2n^2 + 1$ is not an A.P.
3. Find the sum of the series $2+3\frac{1}{2}+5+6\frac{1}{2}+.........+25$ terms.
4. Find the sum of first 15 terms of an AP whose nth term is 9–5n.
5. The 7th term of an AP is 32 and its 13th term is 62. Find the AP.
6. How many terms of the AP 3,5,7.........must be taken so that the sum is 120?
7. If (k+1), 3k, (4k+2) be any consecutive terms of an AP then find the value of k.
8. Show that the sum of the n odd natural numbers equals $n^2$.
9. Insert six arithmetic means between 15 and -13.
10. Determine the number of term in the A.P. 3, 7, 11 ...... 407. Also, find its $20^{th}$ term from the end.

# EXERCISE-II

1. In an AP. Prove that $a_{m+n} + a_{m-n} = 2a_m$
2. The sum of n terms of three Arithmetical progression are $S_1$, $S_2$, $S_3$. The first term of each is unity and common difference are 1,2,3 respectively. Prove that $S_1 + S_3 = 2S_2$
3. If fifth term of an AP is zero. Show that its 33rd term is four times its 12th term.
4. Sum of the first p,q,r terms of an AP are a,b,c respectively. Prove that $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$
5. If the sum of first n terms of an AP is given by $S_n = 3n^2 + 2n$, find the nth term of AP
6. If 'n' arithmetic means are inserted between 20 and 80 such that the ratio of first mean to the last mean is 1 : 3. The find the value of n.
7. Show that the sum of the first n even natural numbers is equal to (1+1/n) times the sum of first n odd natural numbers.
8. Sum of the series $\frac{1}{1+\sqrt{x}},\frac{1}{1-\sqrt{x}},\frac{1+3\sqrt{x}}{1-x}.......$ to n terms.
9. The sum of three numbers in AP is 27 and their product is 504, find them
10. The ratio of the sum of m and n terms of an AP is $m^2 : n^2$. Show that the ratio of the mth and nth term is (2m–1) : (2n–1).
11. If the sum of an A.P is the same for p as for q terms show that its sum for p +q terms is zero.
12. If a, b, c are in A.P. prove that the following are also in A.P.

(i) $\frac{1}{\sqrt{b}+\sqrt{c}},\frac{1}{\sqrt{c}+\sqrt{a}},\frac{1}{\sqrt{a}+\sqrt{b}}$ (ii) $a^2(b+c),b^2(c+a),c^2(a+b)$

13. The 8th term of an AP is zero. Prove that its 38th term is triple of its 18th term.
14. If $a^2 (b + c)$, $b^2(c + a)$, $c^2(a + b)$ are in A.P., show that either a, b, c are in A.P. or ab + bc + ca = 0
15. If there are (2n +1) terms in A.P., then prove that the ratio of the sum of the odd terms and the sum of even terms is (n + 1) : n.

# EXERCISE-III

## SECTION-A

- **Fill in the blanks**

1. If the sum to n terms of a series is $5n^2 + 2n$ then the second term is____________.
2. The solution of the equation $(x+1) + (x+4) + (x+7) + ........ + (x + 28) = 155$ is given by $x$ =_________.
3. The sum to n terms of an AP series is $n^2$. Then the common difference is __________.

## SECTION-B

- **Multiple choice question with one correct answers**

1. If arithmetic mean of a and b is $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ then the value of n is

(A) –1 (B) 0 (C) 1 (D) None

2. $\frac{S_n}{S_m} = \frac{n^4}{m^4}$ (where $S_k$ is the sum of first k terms of an AP $a_1$ $a_2$ $a_3$......∞) then the value of $\frac{a_{m+1}}{a_{n+1}}$ in terms of m and n will be

(A) $\left(\frac{2m+1}{2n+1}\right)^3$ (B) $\left(\frac{2n+1}{2m+1}\right)^3$ (C) $\left(\frac{2m-1}{2n+1}\right)^3$ (D) $\left(\frac{2m+1}{2n-1}\right)^3$

3. If $a_1$, $a_2$, $a_3$.........an are in AP where $a_i > 0$ and for all $i$ then the value of

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + ......... + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}$$

(A) $\frac{1}{\sqrt{a_1}+\sqrt{a_n}}$ (B) $\frac{1}{\sqrt{a_1}-\sqrt{a_n}}$ (C) $\frac{n}{\sqrt{a_1}-\sqrt{a_n}}$ (D) $\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$

4. 8th term of the series $2\sqrt{2}, \sqrt{2}, 0$........... will be

(A) $-5\sqrt{2}$ (B) $5\sqrt{2}$ (C) $10\sqrt{2}$ (D) $-10\sqrt{2}$

5. If the ratio of the sum of n terms of two APs is (3n–13) : (5n+21), then the ratio of 24th terms of the two progression is

(A) 2 : 3 (B) 2 : 1 (C) 1 : 2 (D) None of these

6. Find the sum of all integers between 50 and 500 which are divisible by 7.

(A) 17966 (B) 1177996 (C) 17766 (D) 17696

7. The sum of all 2 digit odd numbers is

(A) 2475 (B) 2530 (C) 4905 (D) 5049

8. If the pth, qth and rth terms of an AP are a,b,c respectively, find the value of a(q–r) + b(r–p) + c(p–q)
(A) 2 (B) 1 (C) 0 (D) 3

9. If $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in AP then
(A) a,b,c are in AP (B) $a^2,b^2,c^2$ are in AP (C) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ are in AP (D) None of these

10. The sum of two numbers is $2\frac{1}{6}$. If an even number of arithmetic means are inserted between them and their sum exceeds their number by 1, then number of means inserted is
(A) 12 (B) 18 (C) 6 (D) None of these

11. If the first term of a series in AP is 17, the last term is $-12\frac{3}{8}$ and the sum is $25\frac{7}{16}$, then find the common difference.
(A) $-\frac{43}{18}$ (B) $-\frac{45}{17}$ (C) $-\frac{47}{16}$ (D) $\frac{47}{16}$

12. If three positive real numbers a,b,c are in AP such that abc = 4, then the minimum value of b is
(A) $2^{1/3}$ (B) $2^{2/3}$ (C) $2^{1/2}$ (D) $2^{3/2}$

## SECTION-C

- **Match the following (one to one)**
**Column-I** and **column-II** contains **four** entries each. Entries of column-I are to be matched with some entries of column-II. Only One entries of column-I may have the matching with the some entries of column-II and one entry of column-II Only one matching with entries of column-I

**1.**

| Column I | Column II |
|---|---|
| (A) Sum of the first 20 terms of the AP –6, 0, 6, 12.......... | (P) 7500 |
| (B) $a_n$ = 4n + 5 is an AP then the sum of the 100 terms in the series | (Q) 1020 |
| (C) Sum of all odd numbers between 100 and 200 | (R) 3050 |
| (D) The sum of integers from 1 to 100 that are divisible by 2 or 5 is | (S) 20700 |

# EXERCISE-IV

## SECTION-A

- **Multiple choice question with one correct answers**

1. If the sum of the first 2n terms of the AP 2,5,8...... is equal to the sum of the first n terms of the AP 57, 59, 61.......then n is equal to
(A) 10 (B) 12 (C) 11 (D) 13

2. Let $T_r$ be the rth term of an AP, for r = 1,2,3.........for some positive integers m, n. We have $T_m$ = 1/n and $T_n$ = 1/m then $T_{mn}$ equals
(A) $\frac{1}{mn}$ (B) $\frac{1}{m}+\frac{1}{n}$ (C) 1 (D) 0

3. A body falls 16 metres in the first second of its motion, 48 m in the second, 80 m in the third, 112 m in the fourth and so on. How for does it fall during the 11th second of its motion?
(A) 338 m (B) 340 m (C) 334 m (D) 336

4. The numbers a,b,c,d,e form an AP then the value of a–4b+6c–4d+e is
(A) 1 (B) 2 (C) 0 (D) None of these

5. The sequence $a_1$, $a_2$, $a_3$, ........., $a_n$ form an AP. Then $a_1^2 - a_2^2 + a_3^2 - a_4^2 + .......... + a_{2n-1}^2 - a_{2n}^2$ is equal to
(A) $\frac{n}{2n-1}\left(a_1^2 - a_{2n}^2\right)$ (B) $\frac{2n}{(n-1)}\left(a_{2n}^2 - a_1^2\right)$ (C) $\frac{n}{n+1}\left(a_1^2 - a_{2n}^2\right)$ (D) None of these

6. If a,b,c,d,e,f are arithmetic mean between 2 and 12, then a+b+c+d+e+f is equal to
(A) 14 (B) 42 (C) 84 (D) None of these

7. If $S_1$, $S_2$ and $S_3$ denotes the sum of first $n_1$,$n_2$ and $n_3$ terms respectively of an AP then

$$\frac{S_1}{n_1}(n_2 - n_3) + \frac{S_2}{n_2}(n_3 - n_1) + \frac{S_3}{n_3}(n_1 - n_2) =$$

(A) 0 (B) 1 (C) $S_1S_2S_3$ (D) $n_1n_2n_3$

8. If a,b,c,d,e,f are in AP then e–c is equal to
(A) 2(c–a) (B) 2(d–c) (C) 2(f–d) (D) (d–c)

9. If the roots of the equation $x^3–12x^2+39x–28 = 0$ are in AP then their common difference will be
(A) ±1 (B) ±2 (C) ±3 (D) ±4

10. If <$a_n$> and <$b_n$> be two sequence given by $a_n = (x)^{1/2n} + (y)^{1/2n}$ and $b_n = x^{1/2n} - y^{1/2n}$ for all n∈ N. Then $a_1a_2a_3$.......$a_n$ is equal to
(A) $x$–y (B) $\frac{x+y}{b_n}$ (C) $\frac{x-y}{b_n}$ (D) $\frac{xy}{b_n}$

## SECTION-B

- **Multiple choice question with one or more than one correct answers**

1. The sum of three integers in AP is 15 and their product is 80. The integers are
(A) 2,8,5 (B) 8,2,5 (C) 2,5,8 (D) 8,5,2

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# *Answers*

## EXERCISE-I

| | | | |
|---|---|---|---|
| **1.** 101 and 3/2 | **3.** 500 | **4.** –465 | **5.** 2,7,12,17... |
| **6.** 10 | **7.** k = 3 | **9.** 11, 7, 3, -1, -5, -9 | **10.** n = 102 & 331 |

## EXERCISE-II

| | | | |
|---|---|---|---|
| **1.** (**) | **2.** (**) | **3.** (**) | **4.** (**) |
| **5.** 6n–1 | **6.** n = 11 | **7.** (**) | **8.** $\frac{n}{2(1-x)}(2+\overline{n-1}\sqrt{x})$ |
| **9.** 4, 9, 14 or 14, 9, 4 | **10.** (**) | **11.** (**) | **12.** (**) |
| **13.** (**) | **14.** (**) | **15.** (**) | |

**NOTE:(**) proof required**

## EXERCISE-III

**Section-A**

| | | |
|---|---|---|
| **1.** 17 | **2.** $x$ = 1 | **3.** d = 2 |

**Section-B**

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | (C) | **2.** | (A) | **3.** | (D) | **4.** | (A) | **5.** | (C) | **6.** | (D) |
| **7.** | (A) | **8.** | (C) | **9.** | (B) | **10.** | (A) | **11.** | (C) | **12.** | (B) |

**Section-C**

**1.** (A)-(Q), (B)-(S), (C)-(P), (D)-(R)

## EXERCISE-IV

**Section-A**

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | (C) | **2.** | (C) | **3.** | (D) | **4.** | (C) | **5.** | (A) | **6.** | (B) |
| **6.** | (A) | **7.** | (B) | **11.** | (C) | **10.** | (C) | | | | |

**Section-B**

**1.** (C,D)

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