

# 11

# CIRCLES

## 11.1 Introduction

## 11.2 Circle

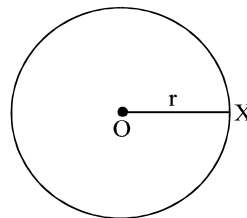
## 11.3 Tangent to a Circle

### 11.1 INTRODUCTION

In class IX, we have discussed about the circle and its centre and radius. Recall that a circle is a collection of all the points in a plane which are at a constant distance (called as radius) from a fixed point (called as centre). We have further studied about the chord, the segment, the sector, an arc etc. related to a circle. In this chapter, we shall discuss about the tangent to a circle and its properties. [The word tangent comes from the Latin word “tangere” which means to touch. It was introduced by the Danish Mathematician Thomas Fimeke in 1583.]

### 11.2 CIRCLE

A circle is a set of all the points in a plane which are at a constant distance from the fixed point. The fixed point is called the **centre** of the circle and the constant distance is called the **radius** of the circle.

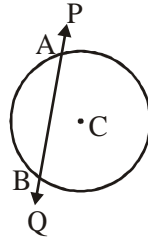


A circle with centre at  $O$  and radius  $= r$  is generally written as  $C(O, r)$ .

A line segment formed by joining the two points on the circle and passing through the centre of circle is called the **diameter** of the circle.

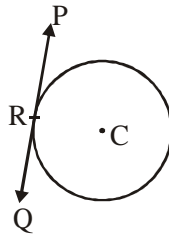
### 11.2.1 Secant

A line which intersects a circle in two distinct points is called a secant of the circle. PQ is a line which intersect a circle two distinct point A and B. PQ is a secant.



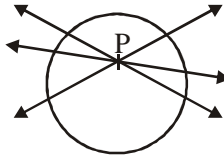
### 11.3 TANGENT TO A CIRCLE

A tangent to a circle is a line that intersects the circle at only one point. In figure PQ is a tangent to a circle and R is called the point of contact of the tangent. The point of intersection of the circle and a tangent to it is known as point of contact.

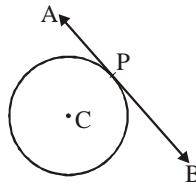


#### 11.3.1 Number of Tangents from a point on a circle

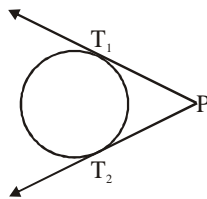
Case-I: There is no tangent to a circle passing through a point lying inside the circle.



Case-II: There is one and only one tangent to a circle passing through a point lying on the circle.



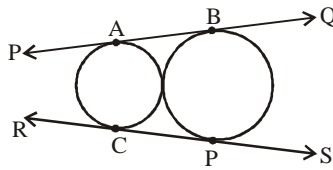
Case-III: There are exactly two tangents to a circle through a point lying outside the circle.



### 11.3.2 Circles and common tangents

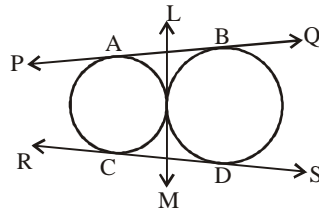
**Case-I:** When circles intersect in two points:

In this case, there will be two common tangents PQ and RS to the two circles as shown in figure.



**Case-II:** When circles touch externally:

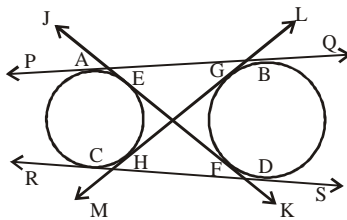
Here the two circles will have three common tangents LM, PQ and RS as shown in figure.



**Case-III:** When one circle lies entirely outside the other circle without having a common point.

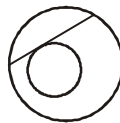
In this case, there will be four common tangents, PQ, RS, JK and LM as shown in figure.

- (i) PQ and RS are two direct common tangents.
- (ii) JK and LM are two indirect common tangents i.e. transversals.



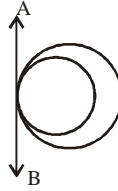
**Case-IV:** When one circle lies entirely inside the other circle without having a common point.

If any tangent is drawn at any point of the inner circle, it will intersect the outer circle in two distinct points and, therefore, cannot be a tangent to the outer circle. Thus, no common tangent can be drawn in this case.



**Case-V:** When circles touch internally

There is only one common tangent AB to the two circles.

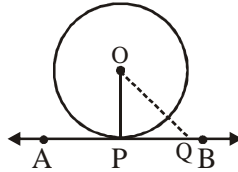


### 10.3.3 Some properties of Tangent to a Circle

**Theorem–1:** The tangent at any point of a circle is perpendicular to the radius through the point of contact.

**Proof:** Let AB be a tangent to the circle with centre O at the point P as shown in the figure. Join OP. We have to prove that OP is perpendicular to AB or AB is perpendicular to OP.

Take a point Q on AB other than the point P. Join OQ. If the point Q lies inside or on the circle, then the line PQ will intersect the circle in two different points and hence a secant.



Which contradicts the tangency of the line.

Therefore, the line Q lies outside the circle.

$\Rightarrow$  OQ is greater than the radius OP, i.e.  $OQ > OP$ . Since it happens for every point on the line AB except the point P.

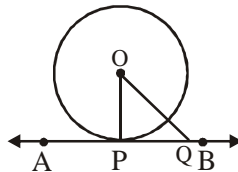
Therefore, out of all the line segments joining the centre to any point on the line AB, the line segment OP is the shortest one.

As we know that among all the line segments joining the point O to a point on the line AB, the shortest one is perpendicular to the line AB.

$\therefore OP \perp AB$  or  $AB \perp OP$ .

Hence, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

**Theorem–2:** A line drawn through the end point of a radius and perpendicular to it is a tangent to the circle.



**Proof:** Let AB be a line perpendicular to the radius OP of a circle with centre at O as shown in the figure.

Take a point Q other than P on the line AB. Since OP is perpendicular to the line AB.

Out of all the line segments joining O to a point on the line AB, OP is the shortest one.

So,  $OP < OQ$  or  $OQ > OP$

The point Q lies exterior to (or outside) the circle.

Every point other than P on the line AB is an exterior point of the circle.

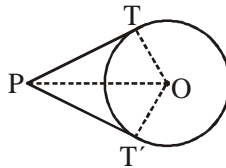
The line AB meets the circle at only one point P.

Hence, the line AB is a tangent to the circle at the point P.

### 11.3.4 Length of Tangent

The length of the segment to the tangent between the point and the given points of contact with the circle is called the length of the tangent from the point to the circle. In figure PT and PT' are the lengths of tangents from point P to the circle.

**Theorem–3:** The lengths of tangents drawn from an external point to a circle are equal.



Proof: Let PT and PT' be the two tangents drawn from a point P lying outside the circle with centre O. Join OT, OT' and OP.

$$\angle OTP = 90^\circ \text{ and } \angle OT'P = 90^\circ$$

In right angled triangles  $\triangle OTP$  and  $\triangle OT'P$ ,

$$\angle OTP = \angle OT'P \quad (\text{Each} = 90^\circ)$$

$$OP = OP \quad (\text{common})$$

$$OT = OT' \quad (\text{radii})$$

By RHS (Right – Hypotenuse –side) congruency criteria.

$$\triangle OTP = \triangle OT'P$$

$$PT = PT' \quad (\text{By cpct})$$

Here, cpct means corresponding parts of congruent triangles.

Hence, the length of tangents drawn from an external point to a circle are equal.

**Remark-1:** It is also clear from  $\triangle OTP \cong \triangle OT'P$

that  $\angle OPT = \angle OPT'$

So, OP is the angle bisector of  $\angle TPT'$  or the centre lies on the bisector of the angle between the two tangents from a point outside the circle.

The theorem-3 can also be proved by using Pythagoras theorem as given below

$$PT^2 = OP^2 - OT^2 = OP^2 - OT'^2 = PT'^2 \quad (OT = OT' = \text{radius})$$

$$\Rightarrow PT = PT'$$

**Normal to a Circle**

The line containing the radius through the point of contact is known as the normal to the circle at the point of contact.

**IMPORTANT RESULTS FOR CIRCLE AND TANGENTS TO A CIRCLE**

1. One and only one tangent can be drawn at any point on the circle.
2. If PAB is a secant to a circle intersecting it at A and B and PT is a tangent, then  $PA \times PB = PT^2$ .
3. The points of intersection of direct common tangents and transverse common tangents to two circles divide the line segment joining the two centres externally and internally respectively in the ratio of their radii.
4. If two chords AB and CD of a circle intersect each other at P outside the circle, then  $PA \times PB = PC \times PD$ .

*Solved Examples*

**Example 1**

*In figure, XP and XQ are tangents from X to the circle with centre O, R is a point on the circle. Prove that,  $XA + AR = XB + BR$ .*

**Solution**

Since lengths of tangents from an exterior point to a circle are equal.

$$\therefore XP = XQ \quad \dots\dots (i)$$

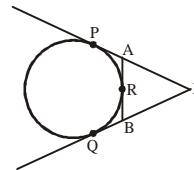
$$AP = AR \quad \dots\dots (ii)$$

$$BQ = BR \quad \dots\dots (iii)$$

Now,  $XP = XQ$

$$\Rightarrow XA + AP = XB + BQ$$

$$\Rightarrow XA + AR = XB + BR \quad [\text{using (i) and (ii)}]$$



**Example 2**

*ABCD is a quadrilateral such that  $\angle D = 90^\circ$ . A circle  $C(O, r)$  touches the sides AB, BC, CD and DA at P, Q, R and S respectively. If  $BC = 38 \text{ cm}$ ,  $CD = 25 \text{ cm}$  and  $BP = 27 \text{ cm}$ , find  $r$ .*

**pdfMachine - is a pdf writer that produces quality PDF files with ease!**  
**Get yours now!**

"Thank you very much! I can use Acrobat Distiller or the Acrobat PDFWriter but I consider your product a lot easier to use and much preferable to Adobe's" A.Sarras - USA

**Solution**

Since tangents to a circle is perpendicular to the radius through the point.

$$\therefore \angle ORD = \angle OSD = 90^\circ$$

It is given that  $\angle D = 90^\circ$ , Also,  $OR = OS$ .

$\therefore$ , ORDS is a square.

Since tangents from an exterior point to a circle are equal in length.

$$\therefore BP = BQ$$

$$CQ = CR$$

and  $DR = DS$

Now,

$$BP = BQ$$

$$BQ = 27 \quad [\because BP = 27 \text{ cm (given)}]$$

$$\Rightarrow BC - CQ = 27$$

$$38 - CQ = 27$$

$$CQ = 11 \text{ cm}$$

$$CR = 11 \text{ cm} \quad [\because CQ = CR]$$

$$\Rightarrow CD - DR = 11$$

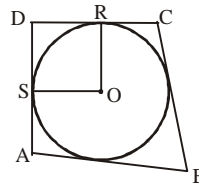
$$25 - DR = 11$$

$$DR = 14 \text{ cm}$$

But, ORDS is a square

$$\therefore OR = DR = 14 \text{ cm}$$

Hence,  $r = 14 \text{ cm}$ .

**Example 3**

*In figure, O is the centre of the circle, PA and PB are tangent segments. Show that the quadrilateral AOBP is cyclic.*

**Solution**

Since tangent at a point to a circle is perpendicular to the radius through the point.

$$\therefore OA \perp AP \quad \text{and} \quad OB \perp BP$$

$$\angle OAP = 90^\circ \quad \text{and} \quad \angle OBP = 90^\circ$$

$$\angle OAP + \angle OBP = 90^\circ + 90^\circ = 180^\circ \quad \dots\dots (i)$$

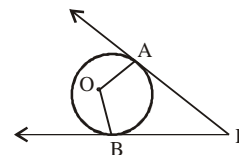
In quadrilateral OAPB, we have

$$\angle OAP + \angle APB + \angle AOB + \angle OBP = 360^\circ$$

$$(\angle APB + \angle AOB) + (\angle OAP + \angle OBP) = 360^\circ$$

$$\angle APB + \angle AOB + 180^\circ = 360^\circ$$

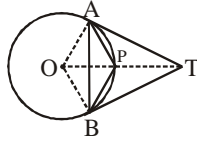
$$\angle APB + \angle AOB = 180^\circ \quad \dots\dots (ii)$$



From (i) and (ii), we can say that the quadrilateral AOBP is cyclic.

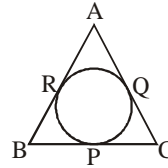
# EXERCISE-I

1. TA and TB are two tangent segments to a circle with centre O from an external point T. If OT intersects the circles in P, prove that AB bisects  $\angle TAB$ .



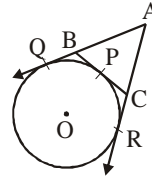
2. The incircle of  $\Delta ABC$  touches the sides BC, CA and AB at P, Q and R respectively. Show that

$$AR + BP + CQ = AQ + CP + BR = \frac{1}{2} \times \text{Perimeter of } \Delta ABC$$

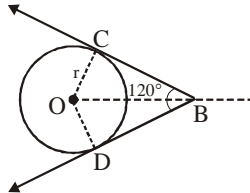


3. A circle of touching the side. BC of a  $\Delta ABC$  at P and touching AB and AC produced at Q and R respectively. Prove that

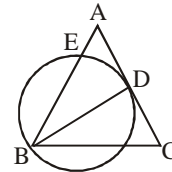
$$AQ = \frac{1}{2} \times (\text{Perimeter of } \Delta ABC)$$



4. Two tangents BC, BD are drawn to a circle C(O,r) such that  $\angle DBC = 120^\circ$ . Prove that  $BO = 2BC$

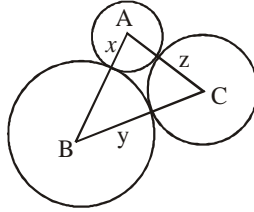


5. ABC is an isosceles triangle such that  $AB = AC$ , D is the mid point of AC. A circle is drawn taking BD as diameter which intersects AB at the point E. Prove that  $AE = \frac{1}{4} AC$

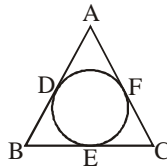




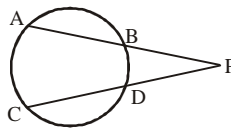
6. Circles are drawn from the three vertices of a  $\Delta ABC$ ; taken as centre to touch each other externally. If the sides of the triangle are 4 cm, 6 cm and 8 cm. Find the radii of the circles.



7.  $\Delta ABC$  is an isosceles triangle with  $AB = AC$ . If the incircle of  $\Delta ABC$  touches the sides  $AB$ ,  $BC$  and  $AC$  at  $D$ ,  $E$  and  $F$  respectively, show that  $E$  bisects  $BC$

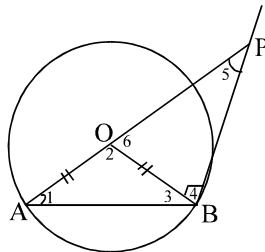


8. Two chords  $AB$  and  $CD$  of a circle intersect each other at  $P$  outside the circle. If  $AB = 5$  cm,  $BP = 3$  cm and  $PD = 2$  cm, find  $CD$ .

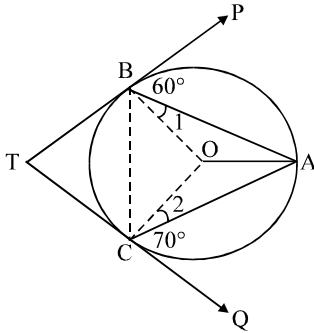


## EXERCISE-II

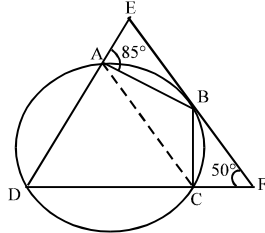
1.  $AB$  is chord of a circle with centre  $O$ . The tangent at  $B$  meets  $AO$  produced at  $P$ , if  $\angle BAP = 35^\circ$  and  $\angle OBP = 90^\circ$ . Find  $\angle BPA$ .



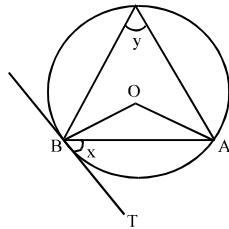
2. In given fig.  $TBP$  and  $TCQ$  are tangents to a circle where centre is  $O$  also  $\angle PBA = 60^\circ$ ,  $\angle ACQ = 70^\circ$  determine  $\angle BAC$  and  $\angle BTC$ .



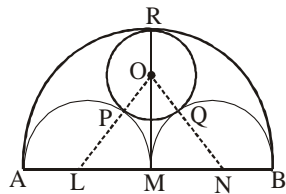
3. In the given fig. ABCD is a cyclic quadrilateral. The tangent to the circle at B meets DC produced at F. If  $\angle EAB = 85^\circ$  and  $\angle BFC = 50^\circ$ , find  $\angle CAB$ .



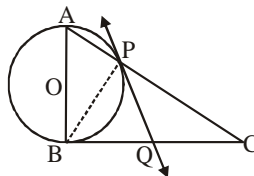
4. In the given fig. AB is a chord of the circle with centre O, BT is tangent to the circle. If  $\angle OAB = 32^\circ$ , find the values of x and y.



5. AB is a line segment and M is its middle point. On one side of AB, semi-circles have been drawn by taking AM, MB and AB as diameters. A circle has been drawn which touches each of the three semi-circles. Prove that the radius of the circle  $r = 1/6 AB$

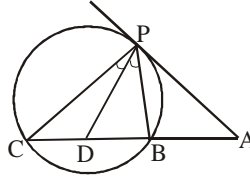


6. In a right angled triangle ABC, a circle with AB as diameter is drawn to intersect the hypotenuse AC in P. Prove that the tangent to the circle at P bisects the side BC.

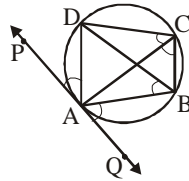


7. In the given figure, AP is a tangent to the circle at P. ABC is a secant such that PD is the bisector of  $\angle BPC$ .

Prove that  $\angle BPD = \frac{1}{2}[\angle ABP - \angle APB]$



8. In a cyclic quadrilateral ABCD, the diagonal CA bisects  $\angle C$ . Prove that the diagonal BD is parallel to the tangent at A to the circle through A, B, C, D.



## EXERCISE-III

### SECTION-A

- **Fill in the blanks**

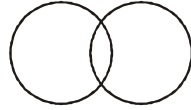
1. Tangent to a circle intersects it in \_\_\_\_\_ points.
2. A line intersecting a circle in two points is called a \_\_\_\_\_.
3. The common point of a tangent to a circle and the circle is called \_\_\_\_\_.
4. A circle can have \_\_\_\_\_ parallel tangent at the most.

### SECTION-B

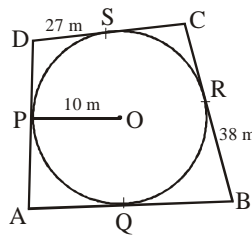
- **Multiple choice question with one correct answers**

1. The locus of the middle points of equal chords of a circle with centre at O is
  - (A) a straight line
  - (B) a circle with centre different with O
  - (C) a circle with centre at O
  - (D) a circle intersecting the given circle at the end of the chord
2. If a regular hexagon is inscribed in a circle of radius r, then its perimeter is
  - (A) 3r
  - (B) 6r
  - (C) 9r
  - (D) 12r
3. ACB is a tangent to a circle at C. CD and CE are chords such that  $\angle ACE > \angle ACD$ . If  $\angle ACD = \angle BCE = 50^\circ$ 
  - (A)  $CE = CD$
  - (B) ED is not parallel to AB
  - (C) ED passes through the centre of the circle
  - (D)  $\triangle CDE$  is a right angled triangle

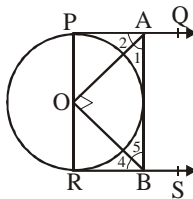
4. Three wires of length  $l_1, l_2, l_3$  form a triangle sumounted by another circular wire. If  $l_3$  is the diameter and  $l_3 = 2l_1$  then the angle between  $l_1$  and  $l_3$  will be  
 (A)  $30^\circ$  (B)  $60^\circ$  (C)  $45^\circ$  (D)  $90^\circ$
5. In the figure two equal circles of radius 4 cm intersect each other such that each passes through the centre of the other. Find the length of the common chord.



- (A)  $2\sqrt{3}$  cm (B)  $4\sqrt{3}$  cm (C)  $4\sqrt{2}$  cm (D) 8 cm
6. In the figure quadrilateral ABCD is circumscribed touching the circle at P,Q,R and S such that  $\angle DAB=90^\circ$ . If CS = 27 cm and CB = 38 cm and the radius of the circle is 10 cm than AB = ?



- (A) 17 cm (B) 28 cm (C) 19 cm (D) 21 cm
7. In the given figure PQ and RS are two parallel tangents to a circle with center O and another tangent AB with point of contact C. Intersects PQ at A and RS at B. Then  $\angle AOB = ?$



- (A)  $90^\circ$  (B)  $60^\circ$  (C)  $30^\circ$  (D) None
8. In the figure, if AD, AE and BC are tangents to the circl at D, E and F respectively. Then  
 (A)  $Ad = AB + BC + CA$  (B)  $2AD = AB + BC + CA$   
 (C)  $4AD = AB + BC + CA$  (D)  $3AD = AB + BC + CA$

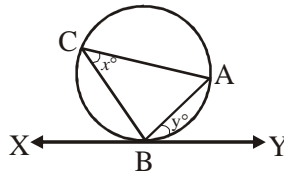
SECTION-C

● **Assertion & Reason**

Instructions: In the following questions as Assertion (A) is given followed by a Reason (R). Mark your responses from the following options.

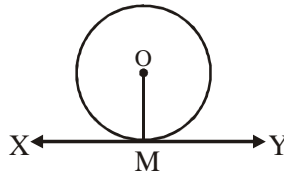
- (A) Both Assertion and Reason are true and Reason is the correct explanation of 'Assertion'
- (B) Both Assertion and Reason are true and Reason is not the correct explanation of 'Assertion'
- (C) Assertion is true but Reason is false
- (D) Assertion is false but Reason is true

1. **Assertion:** If  $x = 40^\circ$  in the following figure then  $\angle y$  is also  $40^\circ$  where XY is a tangent.



**Reason:** Exterior angle of a triangle is equal to sum of its opposite interior angle.

2. **Assertion:** If XY is the tangent of a circle and O is the centre of the circle. Then  $OM \perp XY$ .



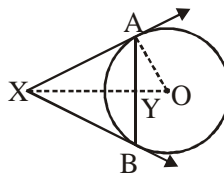
**Reason:** Shortest distance of a point from a given line is the perpendicular distance.

**SECTION-D**

● **Match the following (one to one)**

**Column-I** and **column-II** contains **four** entries each. Entries of column-I are to be matched with some entries of column-II. Only One entries of column-I may have the matching with the some entries of column-II and one entry of column-II Only one matching with entries of column-I

1. If AB is a chord of length 6 cm of a circle of radius 5 cm the tangents at A and B intersect at a point X. The match the column



**Column I**

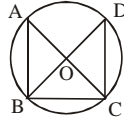
- (A) AY
- (B) OY
- (C) XA
- (D) OA

**Column II**

- (P) 4 cm
- (Q) 3.75 cm
- (R) 5 cm
- (S) 3 cm



5. In the adjoining figure, O is the centre of the circle and  $m(\angle CBD) = 30^\circ$ . Then,  $m(\angle BAC)$  is:

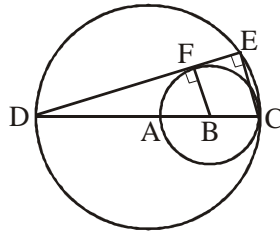


- (A)  $45^\circ$                       (B)  $30^\circ$                       (C)  $60^\circ$                       (D)  $75^\circ$
6. AB and CD are two chords of a circle such that  $AB = 8$  cm,  $CD = 10$  cm and  $AB \parallel CD$ . If the perpendicular distance between AB and CD is 2 cm, then what is the radius of the circle equal to?
- (A)  $\frac{(5\sqrt{17})}{4}$  cm    (B)  $\frac{(4\sqrt{17})}{5}$  cm            (C)  $\frac{(3\sqrt{17})}{5}$  cm    (D)  $\sqrt{17}$  cm

**SECTION-B**

• **Comprehension**

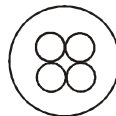
In the diagram DC is a diameter of the large circle centered at A, and AC is a diameter of the smaller circle centered at B. If DE is tangent to the smaller circle at F and  $DC = 12$ .



- Find the length of DE  
 (A)  $8\sqrt{2}$  cm                      (B) 10 cm                      (C)  $7\sqrt{2}$  cm                      (D)  $6\sqrt{2}$  cm
- Find the length of BF?  
 (A) 4 cm                      (B) 5 cm                      (C) 3 cm                      (D) 6 cm
- Find the area of triangle BDF?  
 (A)  $9\sqrt{2}$  cm<sup>2</sup>                      (B)  $3\sqrt{2}$  cm<sup>2</sup>                      (C) 161 cm<sup>2</sup>                      (D) None of these

**AREA RELATED TO CIRCLE**

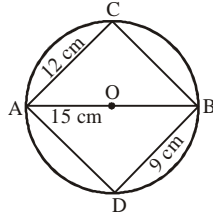
- A path of width 2 meters runs around a circular plot whose circumference is  $75\frac{3}{7}$  metres. Find (i) the area of path. (ii) the cost of gravelling the path at Rs 7 per square meter.
- From a circular sheet of paper with a radius 20 cm, four circles of radius 5 cm each are cut out. What is the ratio of the uncut to the cut portion:



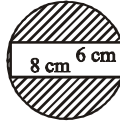
- AB is the diameter of the given circle, while point C and D lie on the circumference as shown. If AB is 15 cm, AC is 12 cm and BD is 9 cm, find the area of the quadrilateral ABCD.

**pdfMachine - is a pdf writer that produces quality PDF files with ease!**  
**Get yours now!**

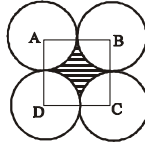
"Thank you very much! I can use Acrobat Distiller or the Acrobat PDFWriter but I consider your product a lot easier to use and much preferable to Adobe's" A.Sarras - USA



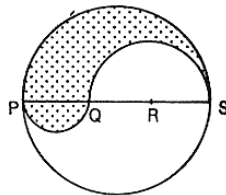
4. The sides of a rectangle inscribed in a circle are 8 cm. and 6 cm. Find the difference of the area of the circle and the rectangle.



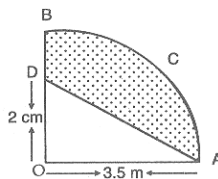
5. ABCD is a square of side 14 cm with centres A, B, C, D four circles are drawn such that each circle touches externally two of the remaining three circles. Find area of the shaded region. (see figure)



6. The radius of a circle is 7 cm. A chord of length  $\sqrt{98}$  cm is drawn in the circle. Find the area of the minor segment.
7. Two circles touch externally. The sum of their areas is  $130\pi$  sq. cm. and the distance between their centres is 14 cm. Find the radii of the circles.
8. A boy is cycling such that the wheels of the cycle are making 140 revolutions per minute. If the diameter of the wheel is 60 cm, calculate the speed per hour with which the boy is cycling.
9. Prove that the area of a circular path of uniform width 'h' surrounding a circular region of radius r is  $\pi h(2r + h)$ .
10. PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters as shown in figure. Find the perimeter and area of the shaded region.

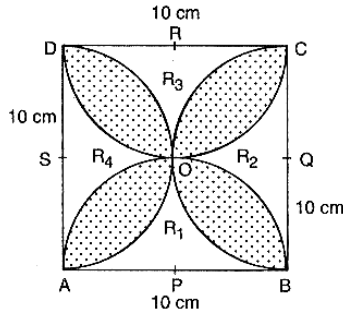


11. In figure, AOBCA represents a quadrant of a circle of radius 3.5 cm with centre O. Calculate the area of the shaded portion (Take  $\pi = 22/7$ )

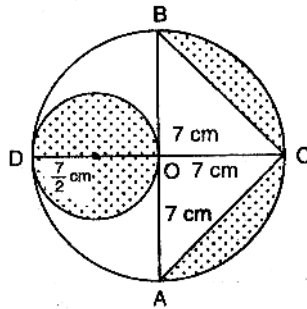




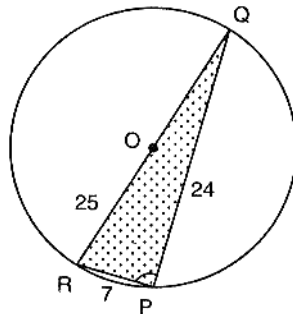
12. It is proposed to add to a square lawn measuring 58 cm on a side, two circular ends. The centre of each circle being the point of intersection of the diagonals of the square. Find the area of the whole lawn.
13. Find the area of the shaded region in figure, where ABCD is a square of side 10 cm (Use  $\pi = 3.14$ )



14. In figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If  $OA = 7$  cm, find the area of the shaded region.



15. Find the area of the shaded region in figure, if  $PQ = 24$  cm  $PR = 7$  cm and O is the centre of the circle.



## *Answers*

### EXERCISE-I

6. 3 cm, 5 cm and 1 cm

### EXERCISE-II

1.  $\angle ABP = 20^\circ$     2.  $\angle BTC = 80^\circ$     3.  $35^\circ$     4.  $x = y = 58^\circ$   
 8.  $CD = 10$  cm

### EXERCISE-III

#### SECTION-A

1. one    2. secant    3. point of contact    4. two

#### SECTION-B

1. (D)    2. (B)    3. (A)    4. (B)    5. (B)    6. (D)  
 7. (A)    8. (B)

#### SECTION-C

1. (B)    2. (A)

#### SECTION-D

1. (A)-(S),(B)-(P),(C)-(Q),(D)-(R)    2. (A)-(Q),(B)-(R),(C)-(S),(D)-(P)

### EXERCISE-IV

#### SECTION-A

1. (B)    2. (D)    3. (A)    4. (C)    5. (C)    6. (A)

#### SECTION-B

1. (A)    2. (C)    3. (A)

### **AREA RELATED TO CIRCLE**

- |  |                         |                           |
|--|-------------------------|---------------------------|
| 1. (i) 1144/7 sq.m.                                      | (ii) Cost = Rs.1144.00  | 2. Ratio = 3 : 1          |
| 3. 108 cm <sup>2</sup>                                   | 4. 30.57cm <sup>2</sup> | 5. 42cm <sup>2</sup>      |
| 6. 14cm <sup>2</sup>                                     | 7. 11cm and 3 cm        | 8. 15.84 km/hr.           |
| 10. Perimeter = $12\pi$ cm. Area = 37.71 cm <sup>2</sup> |                         | 11. 6.125 cm <sup>2</sup> |
| 12. 4325.14cm <sup>2</sup>                               | 13. 57 cm <sup>2</sup>  | 14. 66.5cm <sup>2</sup>   |
| 15. 4523/28 cm <sup>2</sup>                              |                         |                           |