13 PROBABILITY

13.1 INTRODUCTION

In our day-to-day life me generally use the words probably or probable or change (s) or likely etc. in many statements such as :

(i) it will probably rain today.

(ii) He is probably right

(iii) India's cricket team has good chances of winning the world cup 2011.

In such statments, me generally use the words. 'probably', 'chances' most probably etc. These words convey the sense that the event is not certain to take place or, in other words, there is uncertainty about the occurrence (or happening) of the event in the question. In this chapter, me shall introduce the concept of probability as a measure of uncertainty.

Probability is a concept which numerically measure the degree of uncertainty and, therefore, of certainty of the occurrence of an event.

13.2 EXPERIMENT

An operation which can produce some well-defined outcomes is known as an experiment.

13.2.1 Random Experiment

If in each trial of an experiment, conducted under identical conditions, the outcomes in not unique, but may be any of the several possible outcomes then such an experiment is known as random experiment.

In a random experiment, the outcome of each trial depends on chance.

Example

Tossing a fair coin, rolling an unbiased die, drawing a card from a well-shuffled pack of cards are all examples of random experiments.

13.3 SAMPLE SPACE

The set of all possible outcomes in a random experiment is called a sample space and it is generally denoted by S.

Each element of a sample space is called a ***sample point***.

Example

A coin is tossed twice. If the second throw results in a tail then a die is thrown. Describe the sample space.

Solution

Clearly, the sample space is given by

S = {HH, TH, HT1, HT2, HT3, HT4, HT5, HT6, TT1, TT2, TT3, TT4, TT5, TT6}

Example

An experiment consists of rolling a die and then tossing a coin once if the number on the die is even. If the number on the die is odd, the coin is tossed twice. Write the sample space for this experiment.

Solution

Clearly, the sample space is given by

S= {2H, 2T, 4H, 4T, 6H, 6T, 1HH, 1HT, 1TH, 1TT, 3HH, 3HT, 3TH, 3TT, 5HH, 5HT, 5TH, 5TT}

13.4 EVENT

Every subset of a sample space is called an event.

Example

In a single throw of a die, we have

Sample space S = {1, 2, 3, 4, 5, 6}.

The event of getting a prime number is given by

E = {2, 3, 5}

Clearly E $\subseteq$ S.

13.4.1 Impossible Event

Let S be a sample space.

Since $\phi \subset$ S, ϕ is an event, called an impossible event.

13.4.2 Sure Event

Let S be a sample space.

Since S $\subseteq$ S, S is an event, called a sure event.

Example

In a throw of a die, we have

sample space S = {1, 2, 3, 4, 5, 6}.

Let E_1 = event of getting a number less than 1.

And, E_2 = event of getting a number less than 7.

Clearly, no outcome can be less thn 1.

$\therefore$ E_1 is an *impossible event*.

Also, each outcome is a number less than 7.

$\therefore$ E_2 is a *sure event*.

13.4.3 Simple Event

An event containing only a single element of the sample space is called a ***simple*** or an ***elementary event***.

13.4.4 Compound Event

An event which is not simple is called a ***compound*** or ***composite*** or ***mixed event***.

Example

In a simultaneous toss of two coins, we have

sample space S = {HH, HT, TH, TT}.

Then. E_1 = event of getting a tail on the coins = {TT}, is a simiple event.

And, E_2 = event of getting at least 1 tail = {HT, TH, TT}, is a compound event.

13.5 MUTUALLY EXCLUSIVE EVENTS

Two events E_1 and E_2 are said to be mutually exclusive if $E_1 \cap E_2 = \phi$.

However, if $E_1 \cap E_2 \neq \phi$ then E_1 and E_2 are called **compatible events**.

Example

(i) In throwing a die, we have S = {1, 2, 3, 4, 5, 6}.

Let E_1 = event of getting a number less than 3.

And, E_2 = event of getting a number more than 4.

Then, E_1 = {1, 2} and E_2)5, 6}.

Clearly, $E_1 \cap E_2 = \phi$.

Hence, E_1 and E_2 are mutually exclusive.

Let E_1 = event of getting a head on the first coin = {HH, HT}

And, E_2 = event of getting a tail on the second coin = {HT, TT}.

Clearly, $E_1 \cap E_2 \neq \phi$.

Hence, E_1 and E_2 are compatible events.

Example

Two dice are rolled. Let A, B, C be the events of getting a sum of 2, a sum of 3 and a sum of 4 respectively. Then, show that

(i) A is a simple event

(ii) B and C are compound events

(iii) A and B are mutually exclusive

Solution

Clearly, we have

A = {(1, 1)}, B = {(1, 2}, (2, 1)}, and C = {(1, 3), (3, 1), (2, 2)}.

(i) sinceA consists of a single sample point, it is a simple event.

(ii) since Both B and C contain more than one sample point, each one of them is a compound event.

(iii) since $A \cap B = \phi$, A and B are mutually exclusive.

Example

From a group of 2 boys and 3 girls, two children are selected at random. Describe the events.

(i) A = event that both the selected children are girls.

(ii) B = event that the selected group consists of one boy and one girl.

(iii) C = event that at least one boy is selected which pairs of events are mutually exclusive?

Solution

Let us name the boys as B_1 and B_2, and the girls as G_1, G_2 and G_3. Then,

$S = (B_1B_2, B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3, G_1G_2, G_1G_3, G_2G_3\}$.

We have

(i) $A = \{G_1G_2, G_1G_3, G_2G_3\}$

(ii) $B = \{B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3\}$

(iii) $C = \{B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3, B_1B_2\}$

Clearly, $A \cap B = \phi$ and $A \cap C = \phi$.

Hence, (A, B) and (A, C) are mutually exclusive events.

13.6 MUTUALLY EXCLUSIVE AND EXHAUSTIVE SYSTEM OF EVENTS

Let E_1, E_2, ..., E_n be subsets of sample space S. Then, we say that

the events E_1, E_2,......' E_n form a mutually exclusive and exhaustive system if

(i) $E_i \cap E_j \neq \phi$ for $i \neq j$, and (ii) $E_1 \cup E_1 \cup .... \cup E_n = S$.

Example

Suppose we draw a card from a well-shuffled pack of 52 cards.

Let E_1, E_2, E_3 and E_4 be the events of drawing a spade, drawing a club, drawing a heart and drawing a diamond respectively.

As the card drawn is necessarily one of the four types, one of these events is sure to occur.

When one of these events occurs then none of the others occur.

Thus, E_1, E_2, E_3, E_4 form a mutually exclusive and exhaustive system of events.

13.7 INDEPENDENT EVENT

Two events E_1 and E_2 are said to be independent if the occurence of one does not depend upon the occurrence of the other.

If the two events are not independent, they are known as dependent events.

Example

Suppose we toss two unbiased coins

Let E_1 = event of getting a head on the first coin.

And, E_2 = event of getting a head on the second coin.

Clearly, the occurrence of a head on the second coin does not depend upon the occurence of a head on the first coin.

$\therefore$ E_1 and E_2 are independent events.

13.8 EQUALLY LIKELY EVENTS

A given number of events are said to be equally likely if none of them is expected to occur in preference to the others.

Example

If we roll an unbiased die, each outcome is equally likely to happen.

If, however, a die is so formed that a particular face occurs most often then the die is biased. In this case, the outcomes are not equally likely to happen.

13.9 PROBABILITY OF AN EVENT

In a random experiment, let S be the sample space and $E \subseteq S$. Then, E is an event.

The probability of occurrences of E is defined as

$$P(E) = \frac{\text{number of outcomes favourable to occurrence of } E}{\text{number of all possible outcomes}}$$

$$= \frac{\text{number of distinct elements in } E}{\text{number of distinct elements in } S}$$

$$= \frac{n(E)}{n(S)}$$

$$\therefore \; P(E) = \frac{n(E)}{n(S)}$$

Example

Three dice are thrown together. Find the probability of getting a total of at least 6.

Solution

In throwing 3 dice together, the nubmer of all possible outcomes is $(6 \times 6 \times 6) = 216$.

Let E = event of getting a total of at least 6.

Then, $\overline{E}$ = event of getting a total of less than 6.

= event of getting a total of 3 or 4 or 5

= {(1, 1, 1), (1, 1, 2), (1, 2, 1), (2, 1, 1), (1, 1, 3), (1, 3, 1), (3, 1, 1), (1, 2, 2) (2, 1, 2), (2, 2, 1)}.

Now, $n(\overline{E}) = 10 \Rightarrow P(\text{not } E) = P(\overline{E}) = \frac{n\overline{E}}{n(S)} = \frac{10}{216} = \frac{5}{108}$

$\Rightarrow P(E) = 1 - P(\text{not } E) = \left(1 - \frac{5}{108}\right) = \frac{103}{108}$.

Hence, the required probability is $\frac{103}{108}$.

*****

EXERCISE-I

1. An unbiased dice is tossed.
   (i) Write the sample space of the experiment.
   (ii) Find the probability of getting a number greater than 4.
   (iii) Find the probability of getting a prime number.
2. An unbiased die is thrown. What is the probability of getting
   (i) an even number (ii) a multiple of 3
   (iii) an even number or multiple of 3 (iv) an even number and a multiple of 3
3. A card is drawn at random from a well shuffled pack of 52 cards. What is the probability of getting black face card ?
4. In a non-leap year, find the probability of getting 53 Tuesdays.
5. A bag contains 9 red, 7 white and 4 black balls. A ball is drawn at random. What is the probability that the ball drawn will be (a) red (b) black (c) white or black
6. What is the probability that a leap year selected at random will contain 53 Sundays.
7. Find the probability of getting a number less than 5 in a single throw of a die.
8. A card is drawn at random from a pack of 52 cards. Find the probability that card drawn is
   (i) red and a King (ii) either red or King
   (iii) neighter Heart nor a King (iv) neither Ace nor a Queen
9. In a single throw of two dice, what is the probability of
   (i) a total of 9? (ii) doublets.?
   (iii) five-six(i.e. one dice comes up with five and the other with six) ?
   (iv) a multiple of 2 on one and a multiple of 3 on the other ?
10. In single throw of two dice, what is the probability of
    (a) P (an odd number on one dice and 6 on the other)
    (b) P (a number$>$4 on each dice)
    (c) P (a total of 11)
    (d) P (getting same number on either dice)
11. In simultaneous toss of two coins, what is the probability of
    (i) two heads (ii) exactly one head (iii) two tails (iv) no tail ?
12. A dice is thrown twice. Find the probability of getting (i) doublets (ii) prime number on each die.
13. Three unbiased coins are tossed. What is the probability that we get
    (i) all heads (ii) two heads (iii) one head (iv) at least one head.
14. A dice is thrown once. Find
    (a) P (an even number) (b) P (a number$\geq$3)
    (c) P (a number $\leq$4) (d) P (a number $<$7)
    (e) P (a number $>$ 6) (f) P (an ace)

15. One card is drawn from a pack of 52 cards, each of the 52 cards being equally likely to be drawn. Find the probability that :
    (i) The card drawn is red (ii) The card drawn is king
    (iii) The card drawn is Ace (iv) The card drawn is red and Queen
    (v) The card drawn is Spade or a Club
    (vi) The card drawn is Jack, Queen, Kind or an Ace
    (vii) The card drawn is Ace of hearts.
16. A bag contains 3 red, 5 black and 7 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is
    (i) white (ii) red (iii) not black (iv) red or white
17. A card is drawn at random from a pack of 52 playing cards. Find the probability that the card drawn is neither an ace nor a king.
18. A card is drawn at random from a well-shuffled pack of 52 cards. What is the probability that the card bears a number greater than 3 and less then 10?
19. What is the probability that in a group of two people, both will have the same birthday, assuming that there are 365 days in a year and no one has his/her birthday on 29th February?
20. Two dice are tossed together. Find the probability of getting a doublet or a total of 6.
21. In a single throw of two dice, find the probability that neither a doublet nor a total of 10 will appear.
22. A natural number is chosen at random from the first 500. What is the probability that the number so chosen is divisible by 3 or 5?
23. A card is drawn at random from a well-shuffled deck of 52 cards. Find the probability of its being a spade or a king.
24. Two cards are drawn at random from a well-shuffled pack of 52 cards. What is the probability that either both are red or both are kings?

EXERCISE-II

1. A dice is thrown. What is the probability that number shown on the dice is
   (i) an even number (ii) a number divisible by 3
   (iii) a number less than or equal to 4 (iv) a number greater than 6.
2. Two coins are tossed. What is the probability of the appearing of
   (i) at most one head (ii) at most two heads ?
3. Three unbiased coins are tossed. Compute the probability of getting :
   (i) at least two heads (ii) at most two heads.
4. Two dice are thrown together.
   (i) Find the probability that the sum of the digits showing on the top faces of the dice is less than 5.
   (ii) Also find the probability of at least 9 as the sum.

5. A letter is drawn at random from the word ERROR.
   (i) Find the probability of drawing each of the different letters in the word.
   (ii) Demonostrate that the sum of these probabilities is 1.
6. From a pack of cards, one card is drawn at random. Find the probability that
   (i) the card drawn is a black card.
   (ii) the card drawn is red and a court card.
   (iii) the card drawn is either black or an ace.

EXERCISE-III

SECTION-A

- **Multiple choice question with one correct answers**

1. How many triangles can be formed by joining the vertices of a heptagon ?
   (A) 49 (B) 35 (C) 71 (D) 25
2. How many dioganls are there in an n-sided polygon (n>3) ?
   (A) ${}^{n}C_2 - n$ (B) ${}^{n}C_2$ (C) ${}^{n(n-1)}C_2$ (D) ${}^{n}C_{n-2}$

Direction (3-7) : Refer to the data below and answer the question that follow.

Mr. and Mrs. X stays in a house along with their seven children. The female to male ratio in the family is 1 : 2

3. Find the probabilitiy that all the children are of same sex :
   (A) 0 (B) $\frac{2}{5}$ (C) $\frac{1}{21}$ (D) $\frac{5!}{7!}$
4. Find the probability that the five old children are boys and the youngest two are girls :
   (A) 0 (B) $\frac{2}{5}$ (C) $\frac{1}{21}$ (D) $\frac{5!}{7!}$
5. What is the probability that 6 of the children are boys and one is a girl ?
   (A) 0 (B) $\frac{2}{5}$ (C) $\frac{1}{21}$ (D) $\frac{5!}{7!}$
6. When two dice are thrown simultaneously, what is probability that there is exactly one 5 ?
   (A) $\frac{4}{36}$ (B) $\frac{5}{18}$ (C) $\frac{6}{23}$ (D) $\frac{7}{24}$
7. When two fair dice are thrown, what is the probability that both the dice show even numbers ?
   (A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{5}$

SECTION-B

- **Match the following (one to one)**

  Column-I and **column-II** contains **four** entries each. Entries of column-I are to be matched with some entries of column-II. Only One entries of column-I may have the matching with the same entries of column-II and one entry of column-II Only one matching with entries of column-I

1.

Column I	Column II
(A) The card drawn is black	(P) 7/13
(B) The card drawn is a queen	(Q) 1/2
(C) The card drawn is black and a queen	(R) 1/26
(D) The card drawn is either black or a queen	(S) 1/13

EXERCISE-IV

SECTION-A

- **Multiple choice question with one correct answers**

1. When 3 fair dice are thrown what is the probabillity that the sum of the numbers is more than or equal to 6?

   (A) $\frac{103}{216}$ (B) $\frac{103}{108}$ (C) $\frac{47}{216}$ (D) None of these

2. Two coins are such that the first has a tail and a head and the second has both heads. One of these coins is tossed and the result is a head. What is the probability that it is the coin with 2 heads ?

   (A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{2}{7}$

3. Three coins are tossed. What is the probability of getting neither 3 Heads nor 3 Tails ?

   (A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

4. If A and B are two events associated with a random experiment such that P(A) = 0.5, P(B) = 0.3 and $P(A \cap B) = 0.2$. Find $P(A \cup B)$.

   (A) 0.4 (B) 0.3 (C) 0.6 (D) 0.9

5. A and B are two mutually exclusive events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$. Find $P(A \cup B)$.

   (A) $\frac{5}{6}$ (B) $\frac{3}{4}$ (C) $\frac{2}{3}$ (D) None of these

6. What is the probability that the sum of two result is not 3 or 4 ?

(A) $\frac{31}{36}$ (B) $\frac{4}{9}$ (C) $\frac{5}{36}$ (D) $\frac{1}{3}$

7. 3 identical dice are rolled. The probability that the same number will appear on each of them is :

(A) $\frac{1}{6}$ (B) $\frac{6}{216}$ (C) $\frac{1}{216}$ (D) $\frac{3}{28}$

SECTION-B

- **Multiple choice question with one or more than one correct answers**

1. Which of the following can n't be the probability of an event

(A) $\frac{2}{3}$ (B) $\frac{7}{5}$ (C) $-\frac{1}{2}$ (D) 15%

2. The probability of getting a number greater than 2 in throwing a die is —

(A) $\frac{2}{3}$ (B) 0.66 (C) $\frac{1}{3}$ (D) 0.33

3. A card is drawn from an ordinary pack of 52 cards and a gambler bets that, it is a spade or an ace, what are the odds against his winning this bet ?

(A) 4 : 9 (B) 7 : 6 (C) 9 : 4 (D) 8 : 5

SECTION-C

- **Comprehension**

Two dice are rolled sinultaneously. Find the probability of

1. Getting a total of 9 ?

(A) $\frac{1}{3}$ (B) $\frac{1}{9}$ (C) $\frac{8}{9}$ (D) $\frac{9}{10}$

2. Getting a sum greater than 9 ?

(A) $\frac{10}{11}$ (B) $\frac{5}{6}$ (C) $\frac{1}{6}$ (D) $\frac{8}{9}$

3. Getting a total of 9 or 11 ?

(A) $\frac{2}{99}$ (B) $\frac{20}{99}$ (C) $\frac{1}{6}$ (D) $\frac{1}{10}$

4. Getting a doublet

(A) $\frac{1}{12}$ (B) 0 (C) $\frac{5}{8}$ (D) $\frac{1}{6}$

5. Getting a doublet of even numbers ?

(A) $\frac{5}{8}$ (B) $\frac{1}{12}$ (C) $\frac{3}{4}$ (D) $\frac{1}{4}$

6. Getting a multiple of 2 on one dice and a multiple of 3 on the other ?

(A) $\frac{15}{36}$ (B) $\frac{25}{36}$ (C) $\frac{11}{36}$ (D) $\frac{05}{06}$

7. Getting the sum of numbers on the two faces divisible by 3 or 4 ?

(A) $\frac{6}{9}$ (B) $\frac{1}{7}$ (C) $\frac{5}{9}$ (D) $\frac{7}{12}$

8. Getting the sum as a prime number ?

(A) $\frac{3}{5}$ (B) $\frac{5}{12}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

9. Getting atleast one 5 ?

(A) $\frac{3}{5}$ (B) $\frac{1}{5}$ (C) $\frac{5}{36}$ (D) $\frac{11}{36}$

SECTION-D

- **Match the following (one to many)**

Column-I and **column-II** contains **four** entries each. Entries of column-I are to be matched with some entries of column-II. One or more than one entries of column-I may have the matching with the same entries of column-II and one entry of column-II may have one or more than one matching with entries of column-I

Three fair coins are tossed simultaneously. In finding the probabilities match the column I with column II

1.

Column I	**Column II**
(A) Getting one head	(P) 7/8
(B) Getting one tail	(Q) 3/8
(C) Getting atleast one head	(R) 0.375
(D) Getting two atleast two heads	(S) 1/2

Answers

EXERCISE-I

1. (i) 6 (ii) $\frac{2}{6}$ (iii) $\frac{3}{6}$

2. (i) $\frac{3}{6}$ (ii) $\frac{2}{6}$ (iii) $\frac{4}{6}$ (iv) $\frac{1}{6}$

3. $\frac{6}{52}$

4. $\frac{1}{7}$

5. (a) $\frac{9}{20}$ (b) $\frac{4}{20}$ (c) $\frac{11}{20}$

6. $\frac{2}{7}$

7. $\frac{4}{6}$

8. (i) $\frac{2}{52}$ (ii) $\frac{28}{52}$ (iii) $\frac{36}{52}$ (iv) $\frac{44}{52}$

9. (i) $\frac{4}{36}$ (ii) $\frac{9}{36}$ (iii) $\frac{2}{36}$ (iv) $\frac{11}{36}$

10. (a) $\frac{6}{36}$ (b) $\frac{4}{36}$ (c) $\frac{2}{36}$ (d) $\frac{6}{36}$

11. (i) $\frac{1}{4}$ (ii) $\frac{2}{4}$ (iii) $\frac{1}{4}$ (iv) $\frac{1}{4}$

12. (i) $\frac{6}{36}$ (ii) $\frac{9}{36}$

13. (i) $\frac{1}{8}$ (ii) $\frac{3}{8}$ (iii) $\frac{3}{8}$ (iv) $\frac{7}{8}$

14. (a) $\frac{3}{6}$ (b) $\frac{4}{6}$ (c) $\frac{4}{6}$ (d) 1

(e) 0 (f) $\frac{1}{6}$

15. (i) $\frac{26}{52}$ (ii) $\frac{4}{52}$ (iii) $\frac{4}{52}$ (iv) $\frac{2}{52}$

(v) $\frac{26}{52}$ (vi) $\frac{16}{52}$ (vii) $\frac{1}{52}$

16. (i) $\frac{7}{15}$ (ii) $\frac{3}{15}$ (iii) $\frac{10}{15}$ (iv) $\frac{10}{15}$

17. $\frac{11}{13}$

18. 6/13 19. $\frac{1}{365}$ 20. $\frac{2}{9}$

21. 2/9 22. 233/500\ 23. 4/13

24. ******

EXERCISE-II

1 (i) $\frac{3}{6}$ (ii) $\frac{2}{6}$ (iii) $\frac{4}{6}$ (iv) 0

2. (i) $\frac{3}{4}$ (ii) $\frac{4}{4}$

3. (i) $\frac{1}{2}$ (ii) $\frac{7}{8}$

4. (i) $\frac{1}{6}$ (ii) $\frac{10}{36}$

5. (i) $P(E) = \frac{1}{5}$, $P(R) = \frac{3}{5}$, $P(O) = \frac{1}{5}$ (ii) sum = 1

6. (i) $\frac{26}{52}$ (ii) $\frac{6}{52}$ (iii) $\frac{28}{52}$

EXERCISE-III

SECTION-A

1.	B	2.	A	3.	A	4.	C	5.	A
6.	B	7.	B						

SECTION-B

1. (A)-(P),(B)-(S),(C)-(R),(D)-(P)

EXERCISE-IV

SECTION-A

1.	B	3.	C	3.	D	4.	C	5.	A
6.	A	7.	B						

SECTION-B

1.	B,C	2.	A,B	3.	C

SECTION-C

1.	B	2.	C	3.	C	4.	D	5.	B
6.	C	7.	C	8.	B	9.	D		

SECTION-D

1. (A)-(Q,R),(B)-(Q,R),(C)-(P),(D)-(S)